CHAPTER 10

Magnetism: Electricity's Traveling Companion

Magnetic forces have been known since iron-containing rocks were found in antiquity. The magnetic compass was known in China in the second century, and later made possible Columbus's visit to America, but it was not until 1600 that Gilbert suggested that the earth was itself a giant magnet. Today current-carrying coils can produce magnetic fields that are much stronger than those of iron magnets. Most of the electric energy that we use comes from the motion of wires in magnetic fields. Magnetic forces drive the motors in our fans and vacuum cleaners. And perhaps the most startling impact on our civilization comes from the interplay of electric and magnetic fields that we call electromagnetic waves.

We play with magnets from the time when we are children. We pick up pins and nails with them and use them to stick notes to the refrigerator. The forces that they exert are more familiar to us than electric forces, and it comes as a surprise to learn that they are more complex.

The magnetic effects of electric currents were discovered in the early part of the nineteenth century by Oersted, and Ampere soon suggested that the magnetism of iron is the result of internal currents. They were called *amperian* currents, but their detection proved elusive.

There was no way to understand the situation in more detail at that time, since electrons were not identified until near the end of the nineteenth century. An additional obstacle was that it is an unforeseen property of electrons, their *spin*, not discovered until a century after Ampere's suggestion, that is responsible for the strong magnetic effects of iron that we call ferromagnetism. Ampere had the right general idea, but the facts showed themselves to be more complicated and also more interesting than could have been envisioned.

Each electron, and to a lesser extent each proton, is a little magnet. In addition, electrons are in orbit around the nuclei. As a result all materials are magnetic to some extent. As with electric properties, the question becomes "Why are we not more often aware of this?" Electric properties tend to cancel because there are just as many positive as negative charges. A similar cancellation happens with magnetic properties when there are many electrons. Surprisingly, the cancellation is sometimes less complete than in the electric case, especially in the materials that we call *magnetic*, of which iron is the most important.

The origin of the magnetism of materials is now well understood in terms of the properties of atoms and electrons. Today magnetism is a vital subject, under intense investigation for its intrinsic interest and for its important applications, such as magnetic recording and magnetic memories.

10.1 Again—force, field, and motion

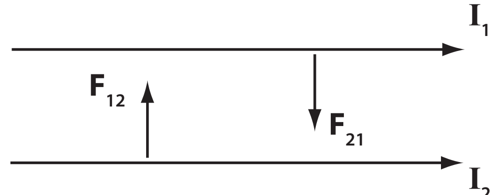
Poles and currents

Electric charges are everywhere, but we rarely think about them or the electric forces between them. Magnetic forces are more familiar. We know them from toys and compasses, and we are aware that the earth is a giant magnet. But what gives rise to the magnetic forces? Is there something like the electric charges that are responsible for electric forces?

Our first experience with this question may be with a compass needle and its north and south poles. If we have two of them, we see that a north pole attracts a south pole and repels another north pole, just as a positive charge attracts a negative charge and repels another positive charge. But as we look further, the situation turns out to be a good deal less simple. If we cut a compass needle in half, we get two new magnets, each with its north and south pole. In fact, all efforts to get an isolated pole have failed.

In some ways the poles act like "magnetic charges," but, as the cutting of a magnet shows, the two kinds cannot be separated. The clue to a more fruitful approach came when Oersted discovered in 1819 that magnets exert forces on electric currents, that is, on moving charges. Soon (in 1825) Ampere suggested that currents play the same role for magnetism that charges play for electricity. That turned out to be the fundamental origin of magnetic effects: magnetic forces are forces between moving charges, over and above the electrostatic (*Coulomb*) forces, and arise when charges are moving with respect to each other.

The force between two parallel currents



If we take two wires, parallel to each other, each with a current in the same direction, we observe that they attract each other. Is that in some ways analogous to the attraction between electric charges with opposite signs and to the gravitational attraction between two objects? Can we use this experiment as a basis for our knowledge of magnetism, just as we used the force law between charges as the basis for our understanding of electricity? The answer is a qualified "yes." There are some similarities, but also important differences.

Currents are not at points, as charges are. We can think of parallel currents as "like" currents and currents in opposite directions as "unlike." Experiments show that like currents attract and unlike currents repel. This is opposite to the force between charges. There are other differences. If we change the separation between the wires with the currents in them, and measure the force of one on the other, we observe that the force is proportional to $\frac{1}{r}$, and not to $\frac{1}{r^2}$, as in Coulomb's law.

We can't really think of a wire with a current as just ending somewhere in the middle of space. In general, there can be a continuous current only in a closed loop, i.e., in a complete circuit. That means that we can't have two straight wires, isolated as in the diagram. As an approximation we can imagine two very long wires, with the rest of the circuit so far away that we don't need to consider it. This becomes the model that we use when we talk of two infinitely long current-carrying wires.

That raises another problem. The longer the wires, with a given current, the greater is the force on each. To separate the dependence on length we need to talk about the force per unit length, $\frac{F_m}{L}$.

We can now combine the observations. The force (F_m) changes with the length of the wires (L), the distance between them (r), and the magnitudes of the currents (I_1 and I_2), so that the magnitude of the force is given by

$$\frac{F_m}{L} = k'\frac{I_1 I_2}{r}$$

k' is the proportionality constant, which in the SI system is 2×10^{-7} N/A^2. We see that it is very small, while the constant in Coulomb's law, $k = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$, is very large. In part this reflects the fact that the *Coulomb* is a very large unit, but also that magnetic forces tend to be much smaller than electric forces. Just as the constant in Coulomb's law can be written as $\frac{1}{4\pi\epsilon_0}$, k′ is often written as $\frac{\mu_0}{2\pi}$, where μ is the Greek *mu*.

EXAMPLE 1

Two parallel wires, each 4 m long, with currents of 4 A in opposite directions, are separated by 5 cm. What is the force on each?

Ans.:

We will assume that the wires are sufficiently long, and the distance between them sufficiently small, that the relation for the force between infinitely long wires gives an adequate approximation.

$F = (4)(2 \times 10^{-7})\frac{(4)(4)}{.05} = 2.56 \times 10^{-4}$ N.

The direction of the force is such that the wires repel each other. We see that although four amperes is a substantial current, the force is quite small.

The magnitude of the magnetic field of a current

The relation for the magnetic force between parallel currents is not as universally useful as Coulomb's law. It describes only what happens in a special situation, and not even one that can actually be realized. Nevertheless we can use it to define the magnetic field.

As in the electric case, we separate the force relation into two parts,

$$\frac{F_m}{L} = \left[k'\frac{I_1}{r}\right][I_2]$$

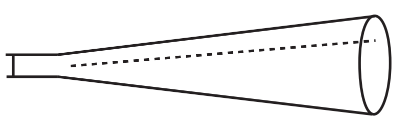
We define the magnitude of the magnetic field by saying that the infinitely long current, I_1, is the source of the magnetic field, $B = k'\frac{I_1}{r}$. The second (finite) current, I_2, with length L, by being in this field, experiences a force, given by $\frac{F_m}{L} = BI_2$, or $F_m = BLI_2$. Just as in the electric case, the magnetic field is introduced only as an aid to calculation, but is later seen to have an independent existence of its own.

The direction of the magnetic field of a current

We still have to define the direction of the magnetic field. This is not as simple as for the electric field. In the electric case there is only a single direction that we have to think about in addition to the field direction, and that is the direction of the force. We let them both lie along the same line, so that there is just one orientation for both field and force.

The magnetic force is an interaction between moving charges, so that there is one more direction that needs to be considered, namely the direction of the current (or moving charge) on which the force acts. That means that we have three directions to keep track of: those of the field, the force, and the current.

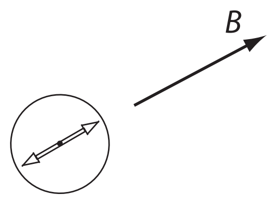
Experiments show that a current in the vicinity of another current experiences a force. That's true for almost all angles between them. There is just one direction of a current for which there is no force on it. This single direction is the one that we use to define the direction of the magnetic field. We do that by defining the field direction so that a current parallel to the field experiences no magnetic force. A current at right angles to the field experiences the maximum force. We'll go on to develop the relation between the size and orientation of the force and the sizes and directions of the magnetic field and the current.



We can do the experiment by actually using a test current whose orientation we can change. To visualize the deflection we can use a small *cathode–ray tube.* This is a vacuum tube with a beam of electrons. (Before their nature was understood, the electrons in such a beam were called cathode rays.) A fluorescent material on the face of the tube lights up (usually green) when hit by the electrons. (Does that sound like a TV tube? Yes. Before the development of flat-panel monitors, all computer and TV screens were in cathode–ray tubes.)

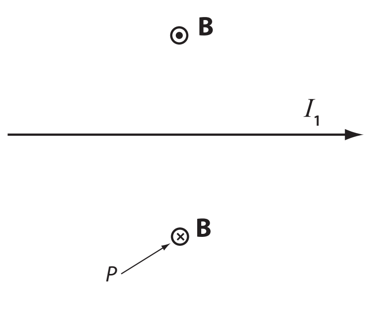
We can move the tube around in the region of a magnetic field and see the deflection of the electron beam. The one direction along which the beam is not deflected is the direction of the magnetic field.

Another way to show the direction of the magnetic field is with a compass needle. A small needle made of iron, free to rotate, will line up along the magnetic field.

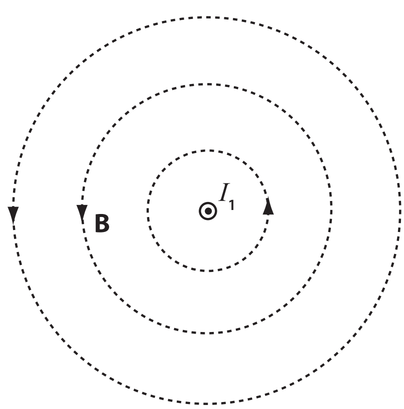


Here, once more, is the current I_1. Surrounding it is its magnetic field B. With the help of observations, using a small cathode–ray tube or a compass needle, we can show that the direction of B is perpendicular to I_1 and tangent to circles whose center is on the wire, as in the head-on view.

The diagram also shows a side view. At the two points that are shown, the magnetic field is perpendicular to the plane of the paper. We represent a vector perpendicular to the paper and into the paper by a circle with a cross (like a feathered arrow from the back) and a vector out of the



side view

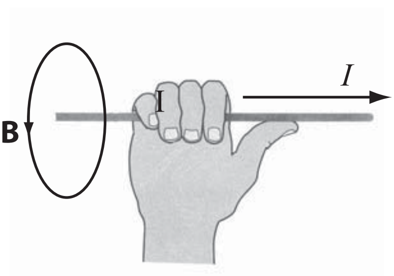


head-on view

paper by a circle with a dot (like an arrow seen head-on). That still leaves us with two possibilities. Is the field direction into the paper or out of it? We choose it to be into the paper at the point *P*.

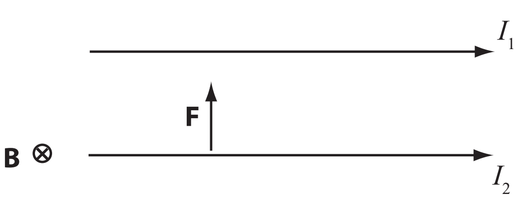
Is that up to us? Doesn't nature tell us what to do? Isn't this supposed to be an operational definition?

The experiment tells us only about the force and its direction. The field is a *construct* that we have invented to help us talk about the force. We can decide its direction, as long as the relation between the field and the force is in accord with the experimental observations.

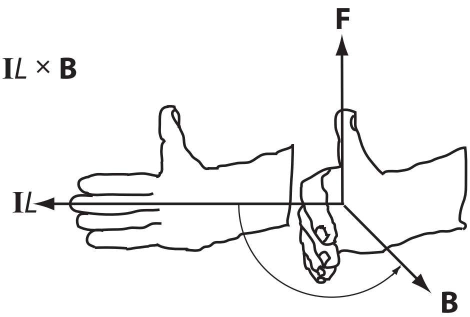


This is what has been agreed on: grasp the current with your right hand, so that your thumb points in the direction of the current. Your fingers then curl in the direction of the field lines. This gives the directions that we show in both views. (We'll call this the first right-hand rule.)

Now let's look at the force on a second current, I_2, in the magnetic field. We already know that when I_2 is in the direction of the magnetic field, it experiences no force, because that's how we defined the direction of the field, and that the maximum force occurs when $\mathbf{I}_2$ and **B** are at right angles to each other. What happens when the magnetic field and the current are at some other angle? In that case we can separate the field into two components. The component parallel to the current does not lead to a force. Only the perpendicular component contributes to the force.



The direction of the field *B* of I_1 at the location of I_2 is *into* the plane of the paper. We can now look at the force on I_2 in that field. From the experimental fact that the two currents attract we know that the force on I_2 is toward I_1.



Here is one of the various rules that have been invented to remind us of the relation between the three directions. Point the fingers of your right hand along the direction of the current. Now curl the fingers so that they point in the direction of the field, letting the fingers of the right hand go from pointing along the current toward pointing along the magnetic field. The thumb then points in the direction of the magnetic force on the current. (We'll call this the second right-hand rule. It relates the directions of the magnetic field, the current, and the magnetic force on the current. The first right-hand rule relates the directions of the current and of the magnetic field that it gives rise to.)

A shorthand way to write a relation that describes both the magnitude and the direction of the force is $\mathbf{F} = I\mathbf{L} \times \mathbf{B}$, where the vector **L** points in the direction of the current.

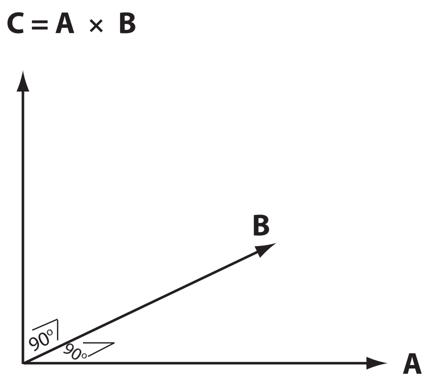
$\mathbf{C} = \mathbf{A} \times \mathbf{B}$ is called the *cross-product* or *vector product* of the two vectors **A** and **B**. Its magnitude is $C = AB \sin\theta$, where θ is the angle between **A** and **B** and the direction of **C** is given by the second right-hand rule.

The cross-product succinctly describes the experimental result: the magnitude $ILB \sin\theta$ is zero when $\sin\theta$ is zero, i.e., when the current and the field are in the same direction. It is at its maximum when $\theta = 90°$, i.e., when the current and the magnetic field are perpendicular.

EXAMPLE 2

(a) What are the units for the magnetic field?

(b) What is the magnetic field of a long wire with a current of 4 A at a distance of 5 cm from it?



Ans.:

(a) From $F = BLI$, the units of B are the same as those of $\frac{F}{LI}$, i.e., $\frac{\text{N}}{\text{A m}}$.

This unit is also called the *tesla* (T). Another unit for B is the *gauss*, equal to 10^{-4} T.

(b) $B = k'\frac{I}{r} = (2 \times 10^{-7})\,\frac{4}{.05} = 1.6 \times 10^{-5}$ T.

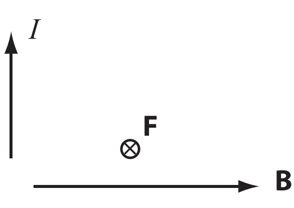
EXAMPLE 3

A wire 30 cm long, carrying a current of 2 A in the y direction, is in a uniform magnetic field of 0.8 T in the x direction. What are the magnitude and direction of the magnetic force on the wire?

Ans.:

The current and the magnetic field are at right angles, so that $F = ILB = (2)(0.3)(0.8) = 0.48$ N.

The direction is given by the second right-hand rule (and by the vector relation $\mathbf{F} = \mathbf{I}L \times \mathbf{B}$). The magnetic force is at right angles both to the current and to the magnetic field, into the plane of the paper.



EXAMPLE 4

For realistic descriptions we have to go beyond infinitely long wires. Most often wires are in closed loops.

A rectangular loop of wire has a width of 20 cm and a length of 30 cm and carries a current of 4 A. It is horizontal and suspended in a uniform magnetic field (0.7 T in the x direction) in such a way that it can turn about an axis through its center, parallel to the long sides.

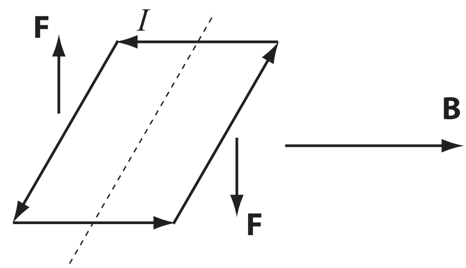
(a) What are the magnetic forces on each of the four sides of the loop?

(b) What is the total magnetic force on the loop?

(c) What is the torque on the loop?

(d) What will be the subsequent motion of the loop?

Ans.:



(a) For a wire perpendicular to the field the magnitude of the force is ILB. Here this is so for the two forces on the longer sides. The magnitude of the force on each is $(4)(0.3)(0.7) = 0.84$ N. The direction of each force is perpendicular both to the field and to the current, so that one is up and the other is down, as shown on the figure. The two short sides are parallel to the field, and there is no force on them.

(b) The two forces are in opposite directions so that the total force on the loop is zero.

(c) Each of the two forces is 0.1 m from the axis of rotation. This is the perpendicular distance from the line of action of the force to the axis. The magnitude of the torque of each is equal to the product of this distance and the force, 0.1 m × 0.84 N or 0.084 N. Both torques act to turn the loop clockwise as seen in the diagram, and add for a total torque of 0.168 Nm.

(d) The loop will start out with an angular acceleration clockwise about the axis (the dotted line in the diagram).

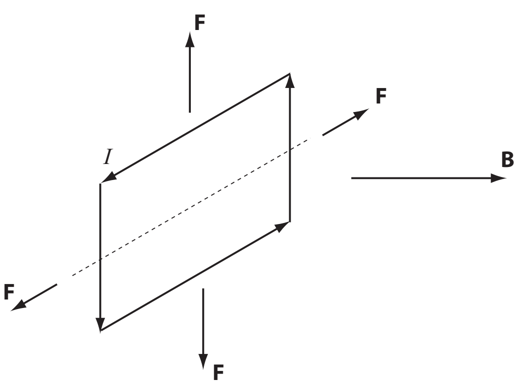
EXAMPLE 5

The same loop as in the previous example is rotated through 90° so that it is at rest perpendicular to the field. Answer the same questions as before.

The orientation of the loop can be described by specifying the direction of the line perpendicular or *normal* to the loop. It is often just called the *normal*. In this example the normal is parallel to the field. Of the two possible directions of the normal we use the one given by a variation of the first right-hand rule: let the fingers of the right hand curl so as to follow the current. The thumb then points in the direction of the normal. Here the normal is in the same direction as the field. In the previous example the normal is perpendicular to the loop and up.

Ans.:

(a) The two long sides are perpendicular to the field. The magnitude of the two forces on them is the same as before. The short sides are also perpendicular to the field. There are now also forces on them, equal to $(4)(0.2)(0.7) = 0.56\,\text{N}$. The directions are shown on the figure. They are such as to tend to stretch the loop.



(b) Both pairs of forces add up to zero, so that the total force is zero.

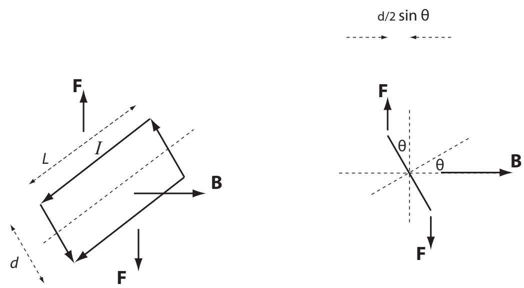
(c) This time the forces on the long sides act along the same line, so that there is no torque. This is also true about the two forces that act on the short sides.

(d) There is no torque and no angular acceleration. The loop is at rest to begin with and remains at rest.

EXAMPLE 6

(a) Write a relation that describes the torque on the loop of the previous examples as a function of the angle that the normal to the loop makes with the magnetic field.

(b) In the position of the previous example there is no torque, and the loop will not continue to turn. What change would cause it to continue to have an angular acceleration and to continue to rotate through another half turn? What would keep it rotating, as in an electric motor?

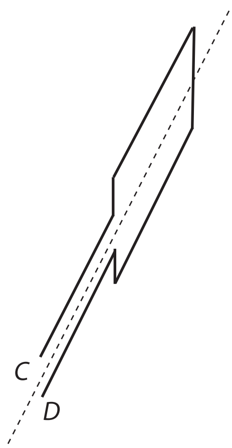


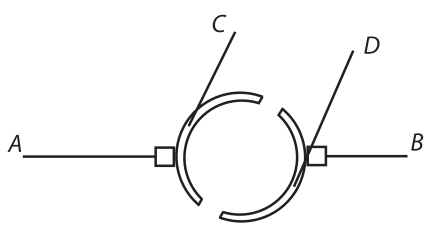
Ans.:

(a) Let the angle between the normal to the loop and the magnetic field be θ. The long sides (of length *L*) are still perpendicular to the magnetic field. The magnitude of the force on each of them is *ILB*.

Let the short sides have length *d*. The forces on them are in opposite directions, and they do not contribute to the torque. The contribution to the torque of each of the forces on the long sides is $(ILB)(\frac{d}{2}\sin\theta)$. The two torques add, to give $ILBd\sin\theta$. Since $Ld = A$, the area of the loop, the magnitude of the torque, τ, is $IAB\sin\theta$. (Both the magnitude and the direction of the torque are given by the vector relation $\tau = I\mathbf{A} \times \mathbf{B}$, where the vector **A** is in the direction of the normal to the loop. The vector τ gives the direction of the torque. It is related to the sense of rotation by a rule analogous to the first right-hand rule. The magnitude of the vector product is $IAB\sin\theta$, where θ is the angle between the vectors **A** and **B**.)

(b) The current would have to be reversed. (It could also be the magnetic field that is reversed, but this is usually more difficult.) The torque would then also reverse, and the loop would turn through another 180°. For continuous rotation, the current, and hence the torque, would have to be reversed every half turn. A built-in switch that makes this happen is called a *commutator*. The diagram shows how it works. A split ring is attached to the axis on which the loop rotates and turns with it. The stationary brushes make contact with the half rings, changing from one to the other each half turn. The current in the loop changes direction every half turn, but the current in the external circuit (connected to the brushes) stays in the same direction. This is the principle of the *DC motor*, i.e., a motor whose current comes from a battery or other source whose emf remains in the same direction.

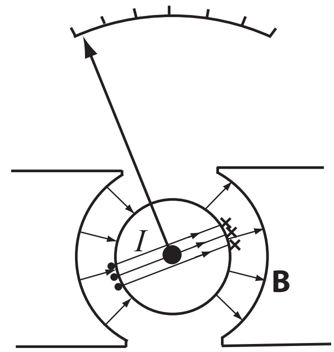




EXAMPLE 7

The three previous examples show what happens to a loop of wire carrying a current when it is in a magnetic field. The interaction between the current and the field results in a torque, and the loop turns. The magnitude of the torque is proportional to the current. The *galvanometer* is a device that uses the torque on a loop to measure the current. The figure shows its essential features.

A wire coil is wound around the cylinder in the middle. The cylinder and coil can turn in the field of a permanent magnet. A needle pointing to a scale is attached to the cylinder. When there is no current in the coil a spring (not shown) fixes the orientation so that the needle points to the left end of the scale.



With a current in it, the loop rotates as a result of the magnetic torque. The spring opposes this motion. The loop turns until the magnitude of the magnetic torque is equal to the opposing torque of the spring. The larger the current, the larger the torque, and the more the coil will rotate. The scale is marked to indicate the magnitude of the current.

For the loops of the previous examples the torque varies as sin θ, where θ is the angle between the normal to the loop and the magnetic field. In a galvanometer we want the magnetic torque to depend only on the current and not on the position of the loop, in other words, not on sin θ. This can be accomplished with a magnet that has curved pole pieces, as on the diagram. The field is then radial, and remains at right angles to the loop as it turns.

(a) Mark the direction of the forces of the magnetic field on the coil.

(b) What is the relation between the torque on the coil and the current through it?

(c) If the torque on the spring follows Hooke's law, what is the relation between the current and the angle through which the coil turns?

Ans.:

(b) $\tau = IAB$.

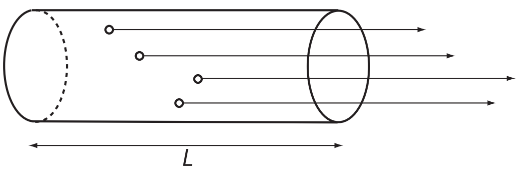
(c) Hooke's law for the spring is $\tau = k\theta$. Hence $k\theta = IAB$. k, A, and B are fixed, so that the angle θ is proportional to the current, I.

Motion of a charged object in a magnetic field

We have talked about electrons moving in wires as an electric current. Can they also move when there is no wire, for example in air? It's not so easy. First they have to be liberated from their "home" atoms. And then they face the obstacle of all the atoms that are in the way and prevent them from moving freely.

Electrons travel through air in sparks and lightning, and after they are emitted from radioactive materials (when they are called beta rays). But they move much more easily when the air is removed, such as in x-ray tubes, vacuum tube rectifiers, cyclotron chambers, and electron microscopes.

One or more charges, moving through empty space, represent a current. Let's see how we can adapt the relation that we have used for the magnetic force on a current for this situation.

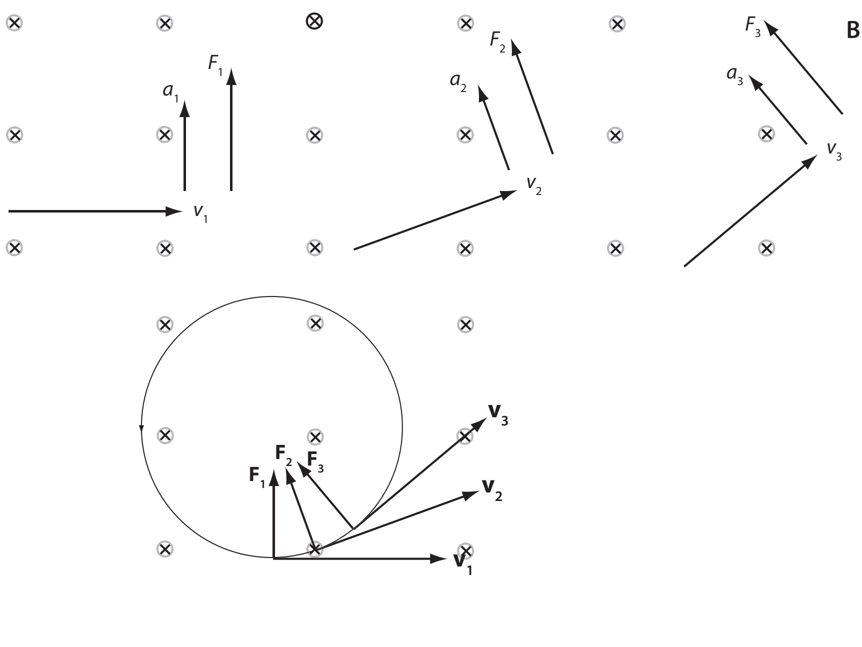


Look at a tube of length L in which charges move at a speed $v = \frac{L}{t}$, so that each takes a time t to traverse the tube. In that time all the charges that were in the tube (a total charge Q) will have left it. The ones that started out at the left-hand end will just make it to the other side. All the others will get further and pass the right-hand end. The current is the rate at which charge passes any cross section. The direction of the current is the same as the direction of the velocity of a positive charge and opposite to the direction of the velocity of a negative charge. It is therefore $I = \frac{Q}{t}$. Hence $Q = It$ and $Qv = Ivt = IL$. We see that we can replace IL in the force law by Qv: the force on a current of length L in a field B perpendicular to it is $F = ILB$, and the force on a charge moving with velocity v is QvB.

These relations apply if the current (or the velocity of the moving charge) is perpendicular to the magnetic field. If a particle moves parallel to the field there will be no force on it and it will continue with constant velocity. The relation that gives the force on the moving charge for all angles between the field and the velocity of the charge is $\mathbf{F} = \mathrm{Q}\mathbf{v} \times \mathbf{B}$. The magnitude of the force is $QvB \sin\theta$, and its direction is given by the second right-hand rule.

The figure shows what happens. The magnetic field is uniform, i.e., its magnitude and direction are the same everywhere. When the velocity is v_1, at right angles to the field, there is a force, and hence an acceleration, at right angles to v_1. The direction of the motion changes. With the velocity changed (in direction but not in magnitude) to v_2, the magnetic force is still at right angles to the field and also to the new velocity. As the charge moves, the velocity vector continues to change direction, and so does the force. The result is that the charge moves in a circle, in a plane perpendicular to the field.

In a magnetic field the magnetic force on a charge is always perpendicular to the direction



of motion of the charge. Hence it can change the direction of the motion, but not the magnitude of the velocity or the particle's kinetic energy.

If the velocity is at an angle to the field, then the component parallel to the field will continue unchanged. The perpendicular component, on the other hand, will cause circular motion in the plane perpendicular to the field. The two together will lead to spiral motion.

We got fairly far, starting with the force between two infinitely long wires, even though we knew from the start that this was a special case. Can we do better? Yes, but at considerable cost in complexity. It is possible to write a relation that describes the contribution to the magnetic field of a tiny ("infinitesimal") piece of current. The contributions from all the pieces can then be added. We won't do that here.

EXAMPLE 8

Circular motion of a charge in a magnetic field

There is a uniform vertical magnetic field of 0.1 T in a region. A charge of 0.1 C enters the field region traveling horizontally with a velocity of 20 m/s.

(a) What is the force on the charge?

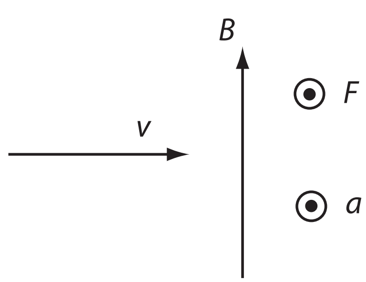
(b) What is the path of the charge? Describe it quantitatively.

Ans.:

(a) For a charge moving at right angles to a magnetic field $F = qvB = (0.1)(20)(0.1) = 0.2$ N.

(b) The path is a circle in the plane perpendicular to the magnetic field. We can find the radius of the circular path by noting that the centripetal force is provided by the magnetic force, i.e., $qvB = \frac{mv^2}{r}$. To calculate the size of the radius we need to know the mass. Let's say it is 10^{-3} kg: $r = \frac{mv}{qB} = \frac{(10^{-3})(20)}{(0.1)(0.1)} = 2$ m.

The direction of the force, and hence of the path, can be found from the right-hand rule.



EXAMPLE 9

A proton whose energy is 1 MeV moves in a circle perpendicular to a magnetic field of 1.2 T.

(a) What is the time, T, for one complete revolution?

(b) What are the frequency, f, (in revolutions per second) and the angular velocity, ω?

(c) How do the quantities T, f, ω, v, and r change when the proton is accelerated to 2 MeV?

Ans.:

(a) The force on a charge in a magnetic field perpendicular to its motion is qvB. Here this is the force that causes the charge to move in a circle, the centripetal force, equal to $\frac{mv^2}{r}$.

We can solve the relation $qvB = \frac{mv^2}{r}$ for the velocity, v, to give $v = \frac{qBr}{m}$.

The velocity times the time for one revolution is equal to the distance that the particle travels in one revolution, i.e., $vT = 2\pi r$, so that $T = \frac{2\pi r}{v}$.

If we now substitute the relation for v in terms of the magnetic field, we see that $T = 2\pi r \frac{m}{qBr}$, which is equal to $\frac{2\pi m}{qB}$. For a proton $m = 1.67 \times 10^{-27}$ kg and $q = e = 1.6 \times 10^{-19}$ C, so that $T = \frac{(2\pi)(1.67\times10^{-27})}{(1.6\times10^{-19})(1.2)} = 5.5 \times 10^{-8}$ s.

(b) The frequency is $\frac{1}{T} = \frac{qB}{2\pi m} = 1.83 \times 10^7$ revolutions per second, or 1.83×10^7 Hz.

The number of radians per second is 2π times the number of revolutions per second, so that the angular velocity is $(2\pi)(1.83 \times 10^7)$ or 1.15×10^8 radians per second.

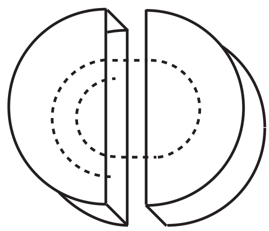
(c) We see that we did not use the fact that the energy is 1 MeV. The time T and the frequency, as well as ω, are independent of the energy! Of course v depends on the particle's energy ($\frac{1}{2}mv^2$) and r is proportional to v.

What are the speed and the radius? Here $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(1)(1.6\times10^{-13})}{1.67\times10^{-27}}} = 1.38 \times 10^7$ m/s.

$r = \frac{mv}{qB} = \frac{(1.67\times10^{-27})(1.38\times10^7)}{(1.6\times10^{-19})(1.2)} = 3.3\times10^{-2}$ m or 3.3 cm.

The proton moves with the same angular velocity in the magnetic field, regardless of its energy. If it could be accelerated each time it crosses a diameter, first to the right and then to the left, with a frequency equal to the frequency of its motion, it could gain energy each time it goes through 180°. This is what happens in a *cyclotron*. The frequency $\frac{qB}{2\pi m}$ is called the *cyclotron frequency*.

The way this is done is that the particles are made to move in the magnetic field in two semicircular metal enclosures (called "dees", because they have the shape of the letter *D*). If an alternating voltage is now applied between the dees, at the cyclotron frequency, the particles will get a kick, i.e., a force and an acceleration along their motion, each time they go through 180°. After each kick from one dee to the other the velocity and hence the radius will increase, so that the particles will spiral outward until they reach the boundary of the magnetic field.

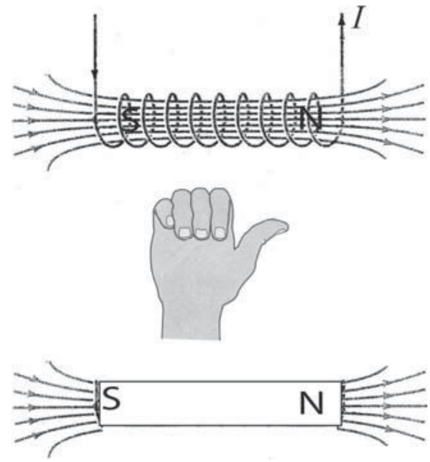


The figure shows the dees and a particle path with an exaggerated acceleration. Inside the dees the path is circular. The voltage between the dees accelerates the particles, so that they move with increased velocity, energy, and radius after they move from one dee to the other. By the time they have moved through a half circle and are about to return to the first dee, the voltage has reversed, and they are again accelerated. (Can you tell what the direction of the magnetic field is?)

Solenoids

A current in a coil gives rise to a magnetic field inside it that is stronger than that of a single loop because the fields of the different loops are roughly in the same direction and add up. A coil used to produce a magnetic field is also called a *solenoid*, meaning "pipeshaped." You can see that the field lines seem to flow through the coil like a liquid through a pipe.

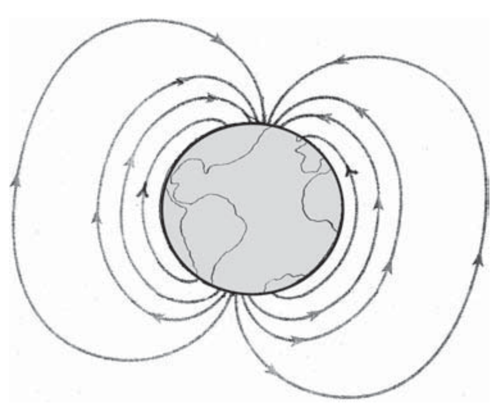
The figure shows the magnetic field of a coil. We see that near the center, some distance from the ends, the field is uniform. We can use the first right-hand rule to relate the direction of the current to that of the magnetic field: grasp the coil with the right hand, with the fingers curling in the direction of the current. The thumb points in the direction of the magnetic field.



The figure also shows a magnet whose size is the same as that of the coil. Its field distribution is the same! Both the coil and the magnet have a north pole from which the magnetic field lines emerge and a south pole where they enter.

The earth's magnetic field

The earth is a giant magnet. It acts as if it had a magnet or a current-carrying coil inside it. In fact, that's just what seems to be the case. The inside of the earth is so hot that part of it is molten, and its rotation gives rise to the earth's magnetic field. Our knowledge of the origin of the earth's field is quite limited. Surprisingly perhaps, the earth's interior is much less accessible to us than the surfaces of the moon, the planets, and even the stars. Most of our knowledge comes from mechanical (sound-like "seismic") waves, generated by natural phenomena, such as volcanic activity and earthquakes.



The magnetic field of a magnet points away from its north pole, which is attracted to the south pole of another magnet. That's where we run into an inconsistency: early on, the earth's pole to which the north pole of a compass needle is attracted was called the north pole. We still call it that, but if we think of the earth as a giant coil or magnet, it is the other way around. What we call the earth's north pole is the south pole of its internal magnet!

Charged particles are part of the "cosmic rays" coming toward the earth. They are deflected by the magnetic field of the earth into spiral paths surrounding the magnetic field lines and form belts of radiation around the earth named after their discoverer, the *Van Allen* belts.

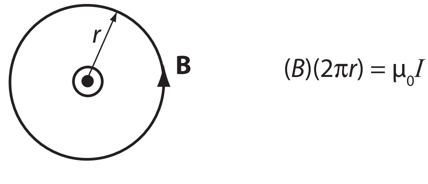
Only when the charged particles come toward the earth parallel to its magnetic field are they undeflected. This happens when they come toward the poles parallel to the earth's axis. Therefore more particles come through the atmosphere there. They ionize the air molecules. When the ions recombine with electrons, radiation is emitted, some of it visible, called the *aurora* (the *aurora borealis* or Northern Lights in the north and the *aurora australis* or Southern Lights in the south).

Ampere's law

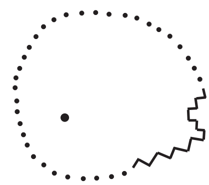
Gauss's law is a general law about charges and electric fields. Is there also a general law for currents and magnetic fields? Yes, there is, but it looks quite different. It is called *Ampere's law*, and we will show what it is in this section. In the next chapter we will see that in this form it is incomplete, and how Maxwell, by extending it, was able to develop the concept of *electromagnetic waves*.

Look again at the magnetic field surrounding a long straight wire and follow one of the circular field lines with radius r. The magnitude of the field is $k'\frac{I}{r}$ or $\frac{\mu_0 I}{2\pi r}$. Now calculate B times the length of the path $(2\pi r)$ as we go around. We get $(B)(2\pi r)$ or $(\frac{\mu_0 I}{2\pi r})(2\pi r)$, which is equal to $\mu_0 I$.

We can go around the wire along other paths, again multiplying B for each piece of path times the length Δs of the piece of path. We can approximate the path by a series of segments parallel and perpendicular to the magnetic field. There is no field radially out from



the wire, only the field tangential to the circular field lines, as before. The result of multiplying each piece of path, Δs, by the magnetic field component parallel to it is therefore again $\mu_0 I$.

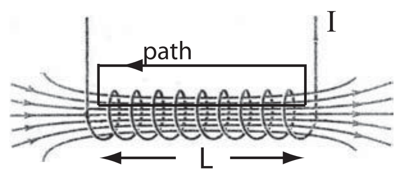


We have taken the paths around a long straight wire, but the result is the same for wires with other shapes. The path times the magnetic field component parallel to it, around a current I, is always $\mu_0 I$. This is *Ampere's law*.

EXAMPLE 10

The field inside a long solenoid (or coil) is uniform, i.e., it has the same magnitude and direction everywhere inside the solenoid. What is its magnitude?

Ans.:

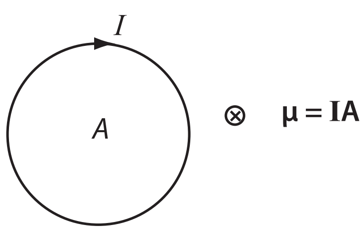


Use Ampere's law. Take a path of length L inside the solenoid and return outside. Inside the path length is L in the field B. For N turns within the path, the enclosed current is NI. The outside part of the path does not contribute, since the field is zero there. The total contribution is therefore BL, and it is equal to $\mu_0 NI$. Hence $BL = \mu_0 NI$ and $B = \mu_0 \frac{N}{L} I$.

10.2 The electron: an old friend turns out to be the elemental magnet

Spin and magnetic moment

We are familiar with the fact that the electron's charge is the fundamental unit of charge. The electron, with its *spin*, is also the fundamental magnet. The quantity that describes its magnetic properties is called its magnetic moment.



Here is the definition of the magnetic moment: take a circular loop of wire with area A carrying a current I. The quantity IA is called its magnetic moment. The symbol usually used for it is μ (Greek *mu*). This is a different use of the symbol μ from the one that we introduced earlier in this chapter. The magnetic moment is a vector quantity whose direction is defined to be perpendicular to the loop. We still have to decide between the two possible directions. Do it as before, using the first right-hand rule: curl the fingers of the right hand in the direction of the current. The thumb then points in the direction of the magnetic moment.

How is it that the electron has a magnetic moment? Where is the loop, where is the current?

The electron's magnetic moment is not associated with a current through a loop, but with another kind of motion, namely the electron's spin. Each electron has spin angular momentum. It is always there, regardless of any other motion of the electron. And the angular momentum and the magnetic moment that is associated with it always have the same size.

A tennis ball, even if it isn't going anywhere, has angular momentum if it spins. That is more or less the situation for an electron, except that you can't stop it from having this angular momentum and you can't change the amount. Both the spin angular momentum and the spin magnetic moment are fixed properties of each electron.

The electron, however, is not a *classical* entity that follows classical rules. It is not a little ball spinning about its axis. One indication of the nonclassical rules that it follows is that it has this intrinsic angular momentum, that it always has it, and always with the same magnitude.

Electron Spin and Its Orientation

The magnitude of the spin angular momentum of the electron is $\frac{\sqrt{3}}{2}\hbar$, where $\hbar$ ("h bar") is Planck's constant, h, divided by 2π. (In a notation that is used for other atomic angular momentum values this can be written $\sqrt{s(s+1)}\hbar$, where $s = \frac{1}{2}$.) The component of this angular momentum along the direction of a magnetic field is $\frac{1}{2}\hbar$.

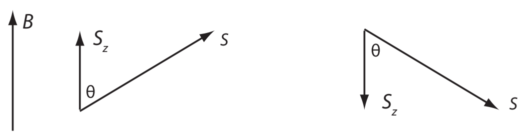
A charge that has angular momentum also has a magnetic moment. The spin magnetic moment has a component along the direction of the magnetic field, either in the direction of the field ("up") or in the opposite direction ("down"). Its magnitude is $\frac{e\hbar}{2m}$ (where e is the electronic charge), and this amount of magnetic moment is called a *Bohr magneton*.

(a) Draw a diagram of the vector **S** representing the spin angular momentum, and its component S_z along the magnetic field. Do this for both possible orientations. What is the angle between **S** and S_z?

(b) Show that the units of the Bohr magneton are those of a magnetic moment.

Ans.:

(a) $S_z = \frac{1}{2}\hbar$, $S = \sqrt{\frac{1}{2}(\frac{1}{2}+1)}\hbar = \frac{\sqrt{3}}{2}\hbar$, and $\cos\theta = \frac{0.5}{0.866}$, $\theta = 54.7°$.



(b) The units of h and $\hbar$ are Js. The units of $\frac{e\hbar}{2m}$ are therefore $\frac{\text{C}}{\text{kg}}$ Js.

J = Nm and N = $\frac{\text{kg m}}{\text{s}^2}$. We can substitute N = $\frac{\text{kg m}}{\text{s}^2}$ to get Js = Nms = $\frac{\text{kg m}}{\text{s}^2}$ m and $\frac{\text{C}}{\text{kg}}$ Js = $\frac{\text{C}}{\text{s m}^2}$, which are the units of IA and therefore the units of a magnetic moment.

The electron is a tiny magnet. It has a magnetic moment, and this is what characterizes a magnet. Its *spin* magnetic moment is responsible for almost all of the strong magnetic properties of matter such as the *ferromagnetism* of iron.

An electron can have additional angular momentum if it is in an atom in orbit about a nucleus. Together with this *orbital* angular momentum there is an *orbital magnetic moment*. The effects of the orbital magnetic moments of atoms are generally smaller than those associated with the spin magnetic moments, and are usually overshadowed by them.

All matter contains electrons, and we can ask why the magnetic properties are only occasionally large. The answer is that it's just as for the charge properties: sometimes two magnetic moments cancel because they are in opposite directions. In other cases magnetic moments may be randomly oriented so that their vector sum is zero. The real surprise is that for some materials the cancellation is not complete. The best-known case is that of iron. An iron atom has a net magnetic moment because the magnetic moment vectors of the electrons in the atom do not cancel.

In a magnetic field atoms with spin magnetic moments tend to line up with their magnetic moments parallel to the field. In some materials the magnetic moments remain aligned even when the external magnetic field is taken away. A piece of material in which this happens is a *permanent magnet*.

The Gyromagnetic Ratio

An object with mass m and charge q is in a circular orbit with radius r at a speed v.

(a) What is its angular momentum, L, in terms of these quantities?

(b) What is its magnetic moment?

(c) What is the magnetic moment divided by the angular momentum? This ratio is called the *gyromagnetic ratio.* Express this quantity in terms of q and m only. (Remember that the current is the charge passing by per second, i.e., the amount of charge passing in a given time divided by that time.)

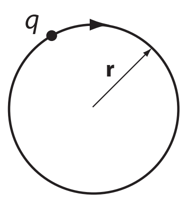
Ans.:

(a) $L = mvr$.

(b) $\mu = iA$, where $A = \pi r^2$. If the time for one revolution is T, the current is $i = \frac{q}{T}$. T is given by $2\pi r = vT$, so that $i = \frac{q}{T} = \frac{q}{2\pi r/v} = \frac{qv}{2\pi r}$. μ is then $(\frac{qv}{2\pi r})(\pi r^2) = \frac{qvr}{2}$.

(c) $\frac{\mu}{L} = \frac{qvr}{2mvr} = \frac{q}{2m}$.

We see that the ratio between the magnetic moment, μ, and the angular momentum, L, of a charge moving in an orbit depends only on the mass and the charge of the orbiting object. In other words, it is independent of the size of the orbit and the speed of the moving charge. This makes it a fundamental and interesting quantity. $\frac{q}{2m}$ is the *classical* value of the gyromagnetic ratio.



$$L = mvr \qquad \mu = I\pi r^2 \qquad \frac{\mu}{L} = \frac{q}{2m}$$

One of the indications that the electron is not a classical object is that it has a spin angular momentum, and that its value is always the same. Another is that the spin's gyromagnetic ratio, the ratio between the spin magnetic moment and the spin angular momentum, does not have the classical value $\frac{e}{2m}$, but that it is instead twice as large, equal to $\frac{e}{m}$.

Magnetic materials

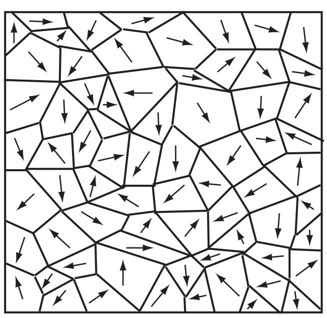
When a material is put in a magnetic field, the atoms and molecules of which it is composed experience forces and torques as a result of their magnetic moments. The magnetic moments may be *intrinsic*, i.e., those that the atoms and molecules have even if there is no magnetic field. In addition, there are always also the magnetic moments that are there because there is a magnetic field and that are absent without it.

The two behave quite differently. An intrinsic magnetic moment (one that is always there, like the spin magnetic moment) experiences a torque that tends to line it up with the field and will strengthen it. This effect is called *paramagnetism*, and a material in which it predominates is *paramagnetic*.

A magnetic moment that is created by a field points in the direction opposite to the field and weakens it. (This is shown by Faraday's law, which is described in the next section.)

If the intrinsic magnetic moments are those of the separate individual atoms, the effects are usually quite small. They can, however, be very large if the atomic magnetic moments interact with each other so strongly that they line up parallel to one another even when there is no external magnetic field. This is what happens in iron and other *ferromagnetic* materials.

The total magnetic moment of a piece of iron is, however, usually small or zero. The material breaks up into *domains*, regions in which the atomic magnetic moments are parallel to each other, but in each domain with a different orientation, so that the net magnetic moment is zero or small. In an external field the magnetic moments of the different domains tend to line up and can enhance the magnetic field by a factor of several thousand.

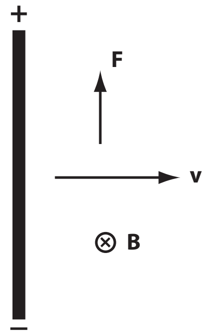


10.3 Generating electricity: motional emf and Faraday's law

The motional emf

We already know that there is a magnetic force on a charge moving in a magnetic field. Can we use this fact to push the electrons in a wire or rod to one side, just as electrons in a battery accumulate at one end?

Look at a metallic rod or wire moving sideways in a magnetic field. Each of the charges in the rod experiences a force, the positive charges toward one end and the negative charges toward the other. The charges separate. The net effect is that one end of the rod becomes positively charged and the other negatively. While it is moving in the magnetic field the rod is *polarized*.

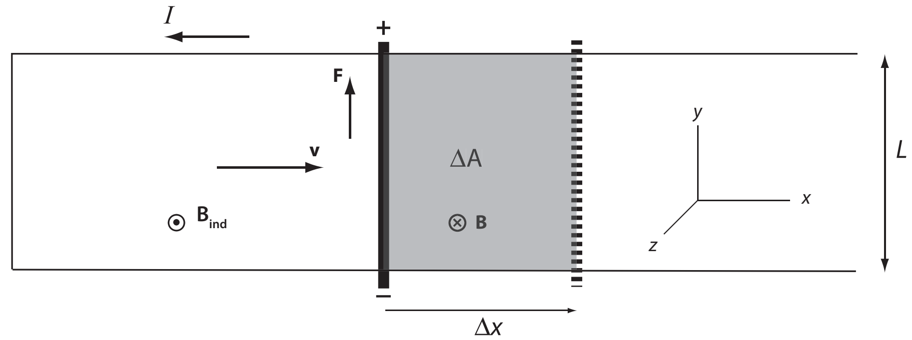


There is now a difference in electric potential, ΔV, between the two ends of the rod. The moving rod acts like a battery. A battery also has two ends, or terminals, with a difference of electric potential between them. The potential difference between the ends of the rod is there as long as the rod continues its motion in the magnetic field.

The difference in electric potential that is created is analogous to the difference in gravitational potential that is produced when an object is lifted. The gravitational potential energy of a lifted object can be changed to kinetic energy when the object returns to its lower position. Here too, the potential energy (this time the electric potential energy) can change to another form when the charges return to where they have lower potential energy.

With a battery this happens when we connect a wire from one terminal to the other. There is then a current from the positive terminal through the wire to the negative terminal. (Since it is the negatively charged electrons that move, their motion is in the opposite direction, from the negative terminal, through the wire outside the battery, to the positive terminal.) As a result of the collisions of the electrons with the ions of the wire the internal energy of the wire increases and the wire heats up. The electric potential energy is transformed to internal ("thermal") energy.

If we do the same with the rod moving in a magnetic field, we have to be careful that the wire through which the charges return is stationary, i.e., not also moving with respect to the magnetic field.



In both the battery and the moving rod there is a transformation of energy to electric potential energy from some other form of energy. In the battery it is the stored internal ("chemical") energy that is liberated by the chemical reactions within it. In the moving rod it is its kinetic energy that is transformed.

The amount of energy that is transformed, divided by the charge that moves, is the *emf*. It represents the energy, for each coulomb that is moved, which is changed to electric potential energy from some other kind of energy.

The figure shows a rod, free to slide from left to right along stationary rails while it continues to make electrical contact with them. There is a magnetic field, B, into the plane of the paper. The rod and the charges within it are being pushed to the right by an external force that is not shown, and they move to the right with velocity v. Because the charges in the rod are moving in a direction perpendicular to the field, there is a force on them, perpendicular both to their velocity and to the field.

The force on an amount of charge, Q, moving with a velocity, v, at right angles to the field B, is QvB. The positive charges experience a force in one direction and the negative charges a force in the opposite direction. The electrons move toward the end marked with a minus sign. The effect is the same as if an equivalent amount of positive charge were to move along the rod in the other direction toward the end marked with a plus sign. If an amount of charge, Q, is moved a distance L by a force, F, the work on it (force times distance) is $FL = QvBL$. The work per unit charge is $\frac{FL}{Q}$, which is equal to BLv. This is the potential difference, or emf, that is created. For this case it is called the *motional induced emf*, $\mathcal{E}$, equal to BLv.

The charges stay separated, and the induced emf continues to exist, as long as the rod or wire continues to move in the field. The figure shows the return circuit. The rod moves along a loop, the rest of which is stationary.

The emf in terms of the change in the flux: Faraday's law

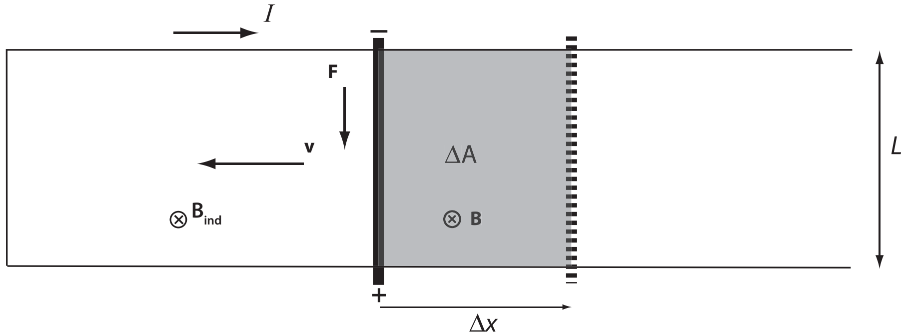
We can now put the relation for the emf in another form which turns out to be much more generally valid. As the wire moves, the area of the loop changes by an amount ΔA, equal to $L\Delta x$ in a time Δt, the change in t.

The rate at which the area changes, $\frac{\Delta A}{\Delta t}$, is equal to $L\frac{\Delta x}{\Delta t}$, or Lv, where v is the rod's velocity. Instead of BLv for the motional emf, we can write $B\frac{\Delta A}{\Delta t}$.

We called the product of the electric field, E, and the area perpendicular to it, the electric flux. Similarly, we call the product of the magnetic field, B, and the area perpendicular to it, the *magnetic flux*, for which we use the symbol Φ (Greek capital *phi*). The magnitude of the motional induced emf can then be written as $\frac{\Delta\Phi}{\Delta t}$.

In our example the velocity, $v = \frac{\Delta x}{\Delta t}$, the rate of change of the area, $\frac{\Delta A}{\Delta t}$, and the rate of change of the magnetic flux, $\frac{\Delta\Phi}{\Delta t}$, are all constant. To encompass the case where this is not so, we use the notation $\frac{d\Phi}{dt}$, representing the rate of change with time of the magnetic flux through the loop, regardless of the particular nature of the variation.

We have not yet considered the direction of the induced emf. Look again at the figure that shows the motional emf. The rod moves to the right (in the x direction) in the external magnetic field, which points into the paper. The area of the current-carrying loop increases by ΔA as



the rod moves. The force on positive charges is along the rod, in the y direction, and there is a current, the *induced current*, counterclockwise around the loop. It gives rise to an *induced magnetic field*, which here is up, out of the paper, in the z direction. (Use the first right-hand rule.) We see that it opposes the *increase* of the original downward magnetic flux.

What happens if the rod moves to the left? Now ΔA and the flux through the loop decrease. The force on positive charges in the rod is in the negative y direction and the other end of the rod becomes positive. The induced current is clockwise. Its magnetic field (the induced magnetic field) is downward, into the paper. It *opposes the decrease* of the original downward flux. It tries to keep the flux there.

In both cases there is a change in the original magnetic flux in the loop. In both cases there is an induced magnetic field that opposes the *change* in the original flux. In the first case it is in the direction opposite to that of the original field. In the second case it is in the same direction. In both cases it opposes the *change* of the original flux. The direction can be incorporated in the statement for the induced emf by using a minus sign: $\mathcal{E} = -\frac{d\Phi}{dt}$.

The statement that the induced field opposes the change in the original field is so important that it is given its own name. It is called *Lenz's law*.

The rod moves in the field so as to produce a magnetic force. Now look at the same event from the reference frame in which the rod is at rest. This time the charges are not moving, so that there is no magnetic force. Nevertheless, experiments show that the emf is still there!

In fact, we would be very surprised if this were not so. We do not expect the laws of physics to depend on the particular reference frame, or coordinate system, from which a phenomenon is observed. More formally, we call this the *principle of relativity*, a cornerstone of the special theory of relativity.

The relation $\mathcal{E} = -\frac{d\Phi}{dt}$ is called *Faraday's law*. It is valid regardless of how the flux changes, regardless, that is, of whether the rod or the loop of wire actually move in a particular coordinate system. It goes farther than the relation for the motional emf, and cannot be derived from the force law.

Faraday's law describes how the motion of wires in a magnetic field can be used to "generate electricity." It shows how the kinetic energy of the wires can be transformed to electric potential energy in an *electric generator*. This is how the overwhelming majority of the electric energy that we use is produced.

EXAMPLE 11

Go to the PhET website (http://phet.colorado.edu) and open the simulation *Faraday's Electromagnetic Lab*.

(a) Choose "bar magnet." Check "show field" and "show compass."

The field is shown by little compass needles. The red end points in the direction of the field. Move the magnet and the compass. See how the compass needle's orientation compares with that of the field vectors. "Flip polarity" and observe the effect.

(b) Choose "pickup coil." Check "show electrons." Move the magnet in and out of the coil and watch the response of the light bulb and of the electrons. Move the magnet slowly to see the direction of motion of the electrons. Increase the area of the loop and observe the effect. A voltmeter can replace the light bulb by clicking on the meter's picture on the right.

When the north pole is moved toward the coil the magnetic flux through the coil increases. What do you observe to be the direction of motion of the electrons? (Clockwise or counterclockwise as seen from the right?) Remember that the motion of the electrons is opposite to the direction of the induced current. What is the direction of the magnetic field (the induced field) produced by the induced current?

What does Lenz's law predict for the induced field? Do the prediction and the observation agree?

As the middle of the magnet comes to the coil, the flux through the coil decreases. What does Lenz's law predict for the direction of the induced field now? What is the direction of the current to produce this field? What is the motion of the electrons? What is your observation for the motion of the electrons? Does it agree with the prediction of Lenz's law?

Here is the answer to part (b). The answers to the other parts follow similarly from Faraday's law and Lenz's law.

As the magnet's north pole enters the loop, the electrons are seen to move CCW so that the induced current is CW and the induced magnetic field is to the left. It counteracts the change in the flux, which here is the increase in the flux to the right.

As the magnet's middle enters the loop, the field and flux decrease. This time the electrons move CW so that the induced current is CCW and the induced field is to the right. It again counteracts the change, which this time means that it counteracts the decrease in the flux. It is to the right, in the same direction as the original flux, so as to counteract its decrease.

(c) Choose "electromagnet." Check "show electrons" and "show compass." Check to see that the relation between the direction of the current and that of the magnetic field is in accord with what you expect.

Select "AC." There are two sliders on the "current supply" box that allow you to change the amplitude and the frequency. Try them out.

(d) Choose "transformer." Check "show field" and "show electrons." Select "DC." Is there an induced current? Explain.

Move the magnet coil (the *primary*) toward the *secondary* (the pickup coil). Predict the direction of the field, the current, and the electron motion induced in the secondary. Which of these quantities can you observe? Are they in accord with your prediction?

Click on "AC" and observe the effect. Look at the light bulb and then switch to the voltmeter. Use the five ways in which the simulation allows you to change the voltage in the secondary. What are they?

(e) Choose "generator." Check "show compass."

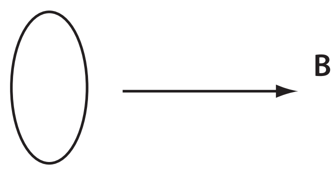
Move the compass near the magnet.

Turn on the "faucet" by turning the knob (drag the small brown cylinder on the knob) to a fairly low frequency (near 30 rpm). Watch the compass, the light bulb, and the electrons. Replace the light bulb by the voltmeter.

There are four ways to change the current in the pickup coil in this simulation. What are they? Try them. There is another that is not available here. What is it?

EXAMPLE 12

A circular loop of wire with an area of 10^{-3} m is perpendicular to a uniform magnetic field of .1 T.



(a) The magnetic field is switched off and smoothly goes to zero in 0.2 s. What is the induced emf in the loop during this time?

(b) The loop is turned through 90° in 0.2 s. What is the induced emf during this time?

(c) The loop is flipped through 180° in 0.2 s, so that it is again perpendicular to the field, but now facing in the opposite direction. What is the induced emf this time?

Ans.:

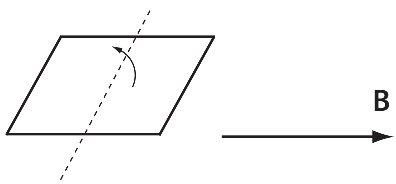
(a) $\Delta\Phi = (0.1)(10^{-3}) = 10^{-4}\,\mathrm{Tm}^2$. $\Delta t = 0.2$. $\frac{\Delta\Phi}{\Delta t} = \frac{10^{-4}}{0.2} = 5 \times 10^{-4}$ V. The direction of the induced emf is such as to produce an induced current with an induced magnetic field that opposes the *change* in the original field. Here it opposes the decrease of the original field and is therefore in the same direction as the original field. The direction of the induced current is related to the direction of the induced magnetic field by the first right-hand rule. This is also the direction of the induced emf.

(b) After the loop is turned, the flux through it is zero. Hence the change of flux is the same as in part (a), and so is the induced emf.

(c) This time the flux, as seen from the loop, changes direction. Sitting on the loop you see the flux changing from $10^{-4}\,\text{Tm}^2$ up to $10^{-4}\,\text{Tm}^2$ down, i.e., a change that is twice as much as before, equal to $\Delta\Phi = 2 \times 10^{-4}\,\text{Tm}^2$. The average induced emf is therefore twice as large as before, or 10^{-3} V.

EXAMPLE 13

emf induced in a turning loop; electric generator



A square loop of wire with sides of 20 cm is suspended so that it can turn about an axis along its centerline. It rotates at a rate of 5 revolutions per second in a uniform magnetic field of 1.2 T.

(a) What is the maximum value of the flux through the loop?

(b) What is the flux as a function of time?

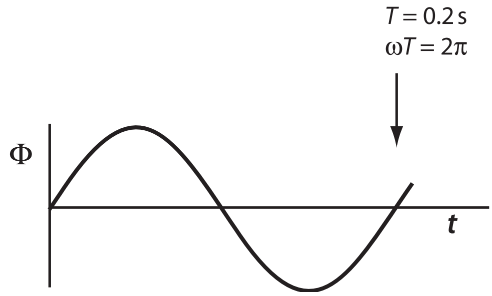
(c) What is the emf induced in the loop?

Ans.:

(a) The flux has its maximum value when the loop is perpendicular to the field. It is then $\Phi_{max} = BA = (1.2)(0.2^2) = 0.048\,\text{Tm}^2$.

(b) Each revolution of the loop turns it through 2π radians. Five revolutions are 10π radians, so that the angular velocity, ω, is 10π radians per second.

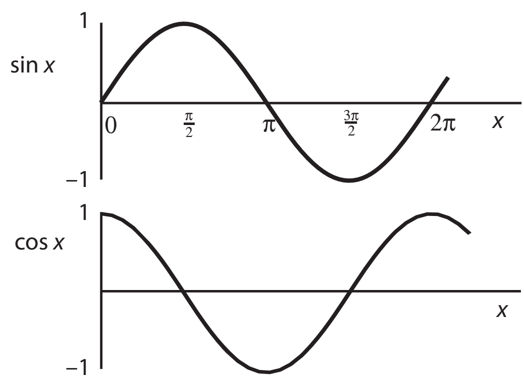
The flux varies between Φ_{max} and zero. It is at its maximum when the loop is perpendicular to the magnetic field and is zero when it is parallel to the field. Two of the sides of the loop remain perpendicular to the field. The others rotate at an angle θ to the field. The flux through the loop is equal to $\Phi = \Phi_{max} \sin\theta$. The angle θ is equal to ωt, so that $\Phi = \Phi_{max} \sin \omega t$.



Here we have assumed that $\Phi = 0$ at $t = 0$, i.e., that at $t = 0$ there is no flux through it at that moment. We could also have used $\Phi = \Phi_{max} \cos \omega t$, which would have the same time variation, but with Φ_{max} at $t = 0$.

(c) The magnitude of the emf is given by $\mathcal{E} = \frac{d\Phi}{dt}$. To calculate the emf we have to know the derivative of $\sin \omega t$.

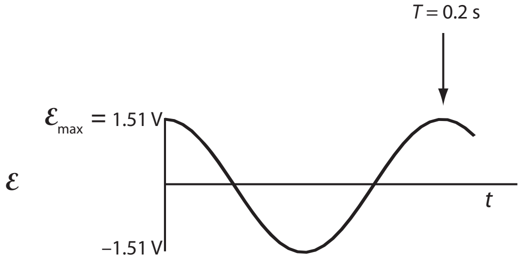
Look first at the derivative of $y = \sin x$. It is $\frac{dy}{dx} = \cos x$.



You can see from the diagram that $\sin x$ has a slope of 1 at $x = 0$, a slope of zero at $x = \frac{\pi}{2}$ and $\frac{3\pi}{2}$, and a slope of -1 at $x = \pi$. These values provide a check on the quoted relation that $\frac{dy}{dx} = \cos x$.

The derivative of $y = \sin \omega t$ is $\frac{dy}{dt} = \omega \cos \omega t$. The factor ω shows that as expected, when the rate of rotation is larger, the rate of change of y is higher.

Finally, we can write the induced emf as $\frac{d\Phi}{dt} = \omega\Phi_{max} \cos \omega t$, equal to $(10\pi)(0.048) \cos 10\pi t$, or $1.51 \cos 34.1t$ V.



The induced emf varies between 1.51 V and −1.51 V at a rate given by the angular velocity of 10π or 34.1 radians/s, i.e., at a frequency ($= \frac{\omega}{2\pi}$) of $5\ \text{s}^{-1}$ or 5 Hz.

We have gone into some detail in this example because of its great importance. An electric generator consists of a coil of wire with many loops, rotating in a magnetic field. This is how almost all of the electricity that we use is generated.

We also see that the induced emf of a rotating coil varies *sinusoidally*. This is one of the reasons why alternating voltage and alternating current (AC) are universally used for household and industrial distribution.

A direct current (DC) generator, i.e., one where the current is always in the same direction, can also be constructed from loops rotating in a magnetic field. However, the direction of the current has to be reversed each time the loop rotates through 180°. This can be accomplished by a *commutator*, like the one that we discussed in connection with the motor, when it was also necessary to reverse the direction of the current after each half turn.

10.4 Summary

The *magnetic force* is an interaction between electric charges over and above the *electric force* between them. It is there when the charges are moving with respect to each other. It can be described either as an interaction between moving charges or as an interaction between currents.

We describe the interaction in two steps: the current (or the moving charge) creates a magnetic field, and another current (or moving charge) in the field experiences a magnetic force.

The magnitude of the magnetic force between two long parallel currents is $\frac{F_m}{L} = k'\frac{I_1 I_2}{r}$. They attract if they are in the same direction and repel if they are in opposite directions.

The magnitude of the magnetic field of a long current, I_1, is $B = k'\frac{I_1}{r}$.

The direction of the magnetic field of a current-carrying wire is given by the first right-hand rule: grasp the wire with your right hand. The fingers curl in the direction of the magnetic field lines.

The magnitude of the force on a current I_2 in a magnetic field is $F = BI_2L$, if B is perpendicular to I_2 and L. If it is not, the relation is still true if for B we use only the component of the magnetic field that is perpendicular to I_2 and L.

The direction of the magnetic force on a current is given by the second right-hand rule: point the fingers of the right hand in the direction of the current, I. Bend the fingers so that they point in the direction of the magnetic field. The thumb then points in the direction of the force.

The shorthand notation that incorporates both the magnitude and the direction of the magnetic force on a current is $\mathbf{F} = I\mathbf{L} \times \mathbf{B}$. In this relation (the *cross product*) the magnitude of $\mathbf{F}$ is $ILB \sin\theta$.

The magnetic torque on a loop with current I and area A is $\tau = I\mathbf{A} \times \mathbf{B}$. (Since the magnetic moment of the loop is $\mu = I\mathbf{A}$, we can also write $\tau = \mu \times \mathbf{B}$.) This is the principle of the electric motor.

The magnetic force on a moving charge is $\mathbf{F} = Q\mathbf{v} \times \mathbf{B}$. If the velocity and the field are perpendicular to each other, $F = QvB$, and the charge moves in a circle in the plane perpendicular to B, with $\frac{mv^2}{r} = QvB$, where $\frac{mv^2}{r}$ is the centripetal force, which here is provided by the magnetic force, QvB.

To determine the direction of the magnetic field of a solenoid coil the fingers in the direction of the current; the thumb points in the direction of B. (First right-hand rule.)

Ampere's law: follow a path around a current. For each step multiply the component of B parallel to the path by the length of the path. The total is $\mu_0 I$. For a circular path $(B)(2\pi r) = \mu_0 I$.

The elementary magnetic object is a current loop, or *magnet*. The electron is a tiny magnet, although there is no loop in that case. The electron (as well as other particles) has angular momentum even when it is not going anywhere. It is called its *intrinsic angular momentum* or *spin*. The magnetism of the electrons leads to the various magnetic properties of materials.

The magnetic moment of a current loop is $\mu = IA$.

The gyromagnetic ratio is the ratio of the magnetic moment to the angular momentum.

A paramagnetic material contains permanent magnetic moments that tend to line up parallel to a magnetic field.

In a diamagnetic material magnetic moments are induced by a magnetic field in accord with Faraday's law. Their direction is opposite to that of the magnetic field.

A ferromagnetic material consists of domains in each of which the magnetic moments are lined up in the same direction. In a magnetic field the domains line up to produce strong magnetic moments.

The vast majority of the electric energy that we use is generated by the motion of wires in magnetic fields. A *motional emf* arises (an electric potential difference appears) between the ends of a wire moving in a magnetic field. A more inclusive and general description is provided by *Faraday's law*.

Motional emf: when a wire of length L moves in a magnetic field, B, with speed v, and L, B, and v are perpendicular to each other, the induced motional emf is $\mathcal{E} = BLv$.

Faraday's law: there is an induced emf in a loop when the magnetic flux through the loop changes. Its magnitude is $\frac{d\Phi}{dt}$.

In a loop the induced emf produces an induced current, which produces an induced magnetic field. The direction of the induced magnetic field is such that it opposes the *change* in the flux, $\Delta\Phi$, that brought it about.

In a coil rotating about its diameter in a magnetic field, the emf varies sinusoidally. This is what happens in an alternating current (AC) generator.

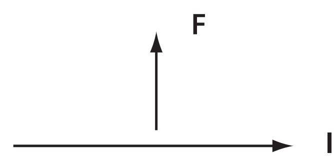
10.5 Review activities and problems

Guided review

1. Two transmission wires are horizontal and parallel to each other, one above the other. The upper one carries a current of 100 A to the right and the lower one carries a current of 60 A in the same direction. How far apart do they need to be if the force on each is to be no more than 10^{-3} N on each meter?

2. A long wire carries a current of 10 A. Where and in what direction does a second, equal and parallel current have to be so that the magnetic field 10 cm from the first wire is zero?

3.

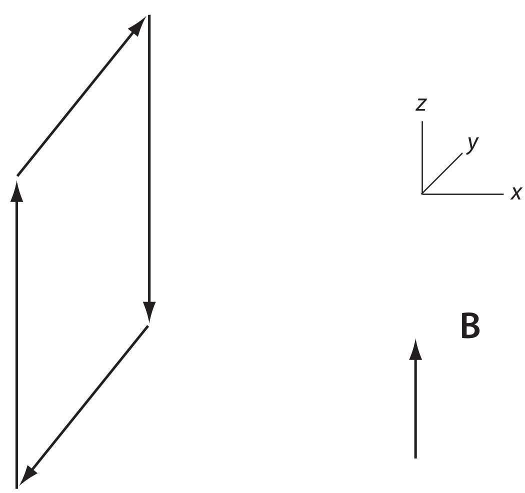


A wire whose length is 50 cm, carrying a current of 8 A in the x direction, experiences a magnetic force of 0.1 N in the y direction.

(a) What are the magnitude and direction of the smallest uniform magnetic field at the wire?

(b) What other magnetic fields at the wire can give rise to the same force?

4.



A square loop (sides 20 cm, $I = 12$ A) lies in the vertical (yz) plane. Looked at from the right, the current is clockwise. There is a uniform magnetic field of 0.5 T in the vertical (z) direction.

(a) What are the forces on each side?

(b) What is the torque on the loop?

5. In what orientation of the loop of the previous question would there be no torque?

6. (a) In what orientation would the loop of the previous questions experience a torque half as large as in question 4? Make a sketch that shows the magnetic field and the loop with its currents and the normal to the loop.

(b) Show the vectors representing the area **A** (in the direction of the normal) and the direction of the vector τ, the torque. Is the direction of the torque as found from the forces the same as that found from the relation $\tau = I\mathbf{A} \times \mathbf{B}$ in this and in the previous two questions?

7. (a) From the forces on the current loop in Example 7, determine the direction of the vector representing the torque exerted by the magnetic field.

(b) Determine the direction of the vector $I\mathbf{A} \times \mathbf{B}$ and compare it to the result of part (a). (The vector **A** is related to the current in the loop by a rule like the first right-hand rule: the fingers of the right hand point along the current and the thumb gives the direction of the area vector.)

8. Looking down, an electron is observed to move clockwise in a horizontal circle. What is the direction of the magnetic field in which it moves?

9. A charged particle moves in a circle in a magnetic field.

(a) Starting with the force relation show that r is proportional to v, and that

(b) the number of revolutions per second is the same for all values of the speed, v.

(c) A cyclotron has a magnetic field of 1 T and a radius of 20 cm. To what energy can it accelerate protons?

10. A coil whose diameter is 5 cm and whose length is 1.2 m is wound with 15,000 turns of wire. What is the magnetic field inside the coil when it carries a current of 20 A?

11. A "flip coil" is used to measure a magnetic field. It is a square loop of wire whose sides are 2 cm. It starts out with its axis (the normal) parallel to the field, and is then flipped through 180° in a time of 0.1 s. The induced emf is observed to be 10^{-3} V. What is the magnitude of the field?

12. In Example 12 the loop is initially perpendicular to the magnetic field, i.e., the normal to the loop is parallel to the field. Now consider the case where the loop is initially parallel to the field (with its normal perpendicular to the field).

(a) The loop turns through 90° in 0.2 s to the position in the figure of the example. What is the average emf during that time?

(b) The loop turns through an additional 90°, again in 0.2 s. What is the emf this time?

(c) What is the average emf when the two motions of parts (a) and (b) are combined, i.e., when the loop turns through 180° in 0.4 s?

13. A loop of wire whose area is 0.2 m^2 rotates in a uniform magnetic field of 0.9 T at a rate of 12 revolutions per second. It starts (at $t = 0$) with its normal perpendicular to the field.

(a) What is the flux through the loop at $t = 0$ and after the loop has rotated through 90°, 180°, and 270°?

(b) Sketch the graphs of Φ and $\mathcal{E}$ as a function of time. On your graphs mark $\Phi_{max}, \mathcal{E}_{max}$, and the time T for one revolution.

Problems and reasoning skill building

1. (a) A positively–charged particle travels horizontally with a velocity v. It enters a region with a uniform magnetic field such that it travels in a horizontal circle. Make a sketch of v, B, and the path with one of the possibilities for the direction of B.

(b) What is the other possibility for the direction of B?

(c) What paths would a negative particle follow in the same fields as in parts (a) and (b)?

2. Two particles with the same mass, traveling with the same velocity, enter a region with a uniform magnetic field, such that they move in circular paths. The first particle has charge q_1 and for the second $q_2 = 2q_1$. What is the ratio $\frac{r_2}{r_1}$ of the radii of the two paths?

3. A charged particle moves in a circle in a uniform magnetic field. An electric field is now turned on, in a direction opposite to that of the

magnetic field. What is the path of the particle now?

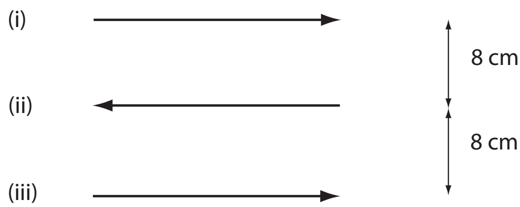
4. A loop of wire is in the plane of the paper. It carries a clockwise current.

(a) What is the direction of the magnetic field at its center?

(b) The area of the loop is $10\,\text{cm}^2$ and the current is 2 A. What are the magnitude and direction of the loop's magnetic moment?

(c) An additional magnetic field, horizontal and to the right, of 0.5 T is turned on. What are the magnitude and direction of the torque on the loop? Through what angle will it turn until the torque is zero? What is the direction of its magnetic moment then?

5. Each of the three long parallel wires (i), (ii), and (iii) carries a current of 5 A.



(a) What is the force on the middle one?

(b) What are the magnitude and direction of the forces per meter on each of the other two?

6. A magnetic field is perpendicular to a loop of wire. Looking down on the loop, what is the direction of the induced current (clockwise or counterclockwise) when the field is

(a) up and steady

(b) up and increasing

(c) up and decreasing

(d) down and increasing

(e) down and decreasing

7. A solenoid is surrounded by a loop of wire. A current is switched on in the solenoid in the clockwise direction when seen from the right.

(a) What is the direction of the induced current? What can you say about the length of time during which there is a current in the loop?

(b) What is the direction of the induced current when the current in the solenoid is switched off?

8. A long straight wire carries a current of $I_1 = 12$ A into the plane of the paper. A second wire is 10 m from the first and parallel to it.

(a) The magnetic field is zero at a point between them in the plane containing both wires, 2 m from the first. What are the magnitude and direction of the current I_2 in the second wire?

(b) Part (a) can be answered by first finding the field of the first current. What is the advantage of not doing that and first developing a relation between I_1 and I_2?

9. A house has a floor area of $120\,\text{m}^3$ and four walls, each of which has an area of $25\,\text{m}^3$. One wall faces north, one south, one east, and one west.

The earth's magnetic field there has a horizontal component of 2.4×10^{-5} T and a vertical component, down, of 4.8×10^{-5} T.

What is the magnetic flux outward through each wall and through the floor? What is the total flux outward through all the walls, the floor, and the ceiling?

10. A circular coil of wire has a radius of 0.15 m and a resistance per unit length of $3.8 \times 10^{-2}\ \Omega/\text{m}$. It is perpendicular to a magnetic field that increases from zero to 0.55 T in 1.5 s. What is the electric energy dissipated in the wire during the 1.5 s?

11. Looking down on its path, a 1 MeV proton travels clockwise with a radius of 10 cm. What are the magnitude and direction of the field in which the proton moves?

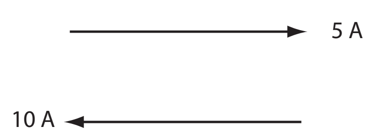
12. A loop of wire whose area is $10^{-2}\,\text{m}^2$ rotates about its diameter in a uniform magnetic field of 0.8 T at a rate of 15 revolutions per second. It starts out (at $\theta = 0$) with its normal parallel to the field.

(a) What is the average emf induced in the loop as it turns through 90° from its starting position?

(b) Repeat part (a) for 180° and 360°.

Multiple choice questions

1.



Two long parallel wires carry steady currents in opposite directions. The current in the upper one is 5 A to the right and the current in the lower one is 10 A to the left. The ratio of the magnetic force on I_1 to that on I_2 is

(a) +1
(b) −1
(c) +2
(d) −2

2. For the same wires and currents as in the previous question, the ratio of the magnetic field at I_1 created by I_2 to that at I_2 created by I_1 is

(a) +1
(b) −1
(c) +2
(d) −2

3. Both uniform magnetic and electric fields can do all but which one of the following on a charged particle?

(a) accelerate it
(b) exert a force on it
(c) change its direction
(d) increase its kinetic energy

4. A proton and an alpha particle ($q = 2e, m = 4m_p$) move with the same speed in circles in a uniform magnetic field. The ratio of their radii $\frac{r_\alpha}{r_p}$ is

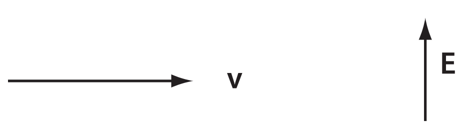
(a) 4
(b) 2
(c) 1
(d) 0.5

5. A proton and an alpha particle move with the same kinetic energy in circles in a uniform magnetic field. The ratio of their radii $\frac{r_\alpha}{r_p}$ is

(a) 4
(b) 2
(c) 1
(d) 0.5

Synthesis problems and projects

1.



An alpha particle travels in the x direction with a velocity of 10^3 m/s. It enters a region with an electric field in the y direction, of 50 N/C.

(a) What are the magnitude and direction of a magnetic field such that the particle continues undeflected?

(b) Replace the alpha particle by an electron, and repeat.

2. An electric generator has 500 turns of wire, each with an area of $0.05\,m^2$. It rotates with a frequency of 60 revolutions per second in a magnetic field of 1.2 T. What is the maximum emf that it produces?

3. In a *mass spectrometer* a particle with mass m and charge q is first accelerated through a potential difference V. It then enters a region where a uniform magnetic field (B) causes it to move in a circle, with radius r.

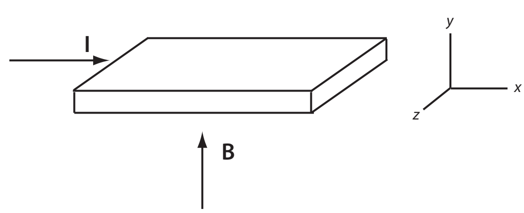
(a) Develop a relation for r in terms of q, m, V, and B.

(b) What is the radius for a proton that is first accelerated through a potential difference of 300 V and then moves in a circle in a magnetic field of 0.05 T?

(c) Use proportional reasoning to find the radius for an alpha particle.

(d) Again using proportional reasoning, by what factor will the radius be different for an electron compared to that for a proton?

4.



A flat horizontal strip carries a current in the x direction. There is a uniform magnetic field in the y direction (up, perpendicular to the strip).

(a) The electrons, moving in the negative x direction, are deflected. What is the direction of the magnetic force on them?

(b) The electrons move toward one side of the strip, which becomes negatively charged, while the opposite side becomes positively charged. This is called the *Hall effect*, and the potential difference at right angles to the current is called the *Hall voltage*.

In addition to the magnetic force at right angles to the current there is now also an electric force. What is its direction?

The build-up of charge continues until the electric force and the magnetic force are equal, so that there is then no further deflection of the electrons as they move through the strip.

(c) Consider the possibility that the current is carried not by electrons but by positive charges. What is the direction of the magnetic force on them?

(In semiconductors the current can be carried by "holes" in the ocean of electrons. The holes act as if they were positive charges. The Hall effect can distinguish between currents carried by holes and by electrons.)

(d) Show how the measurement of the Hall voltage can be used to determine the average velocity of the electrons. (The electrons have large velocities. When there is no current they move randomly in all directions, colliding with the ions of the crystalline lattice. Their average velocity is then zero. When there is a current there is a much smaller additional component of their velocity in the direction opposite to the current. Its average is called the *drift velocity*.)

5. A transformer consists of a primary coil surrounded by a secondary coil. An alternating current in the primary coil induces an alternating current in the secondary coil.

(a) For one quarter cycle the primary current is in the clockwise direction and increases. What is the direction (clockwise or counterclockwise) of the secondary current during that time?

(b) For the next quarter cycle the primary current is in the same clockwise direction and decreases. What is the direction of the secondary current in this part of the cycle?

(c) Describe the direction of the primary and secondary currents and how they change for the following two quarter cycles.

(d) Sketch the primary current as a function of time and underneath it the secondary current. (Use clockwise as positive and counterclockwise as negative.) Write down Faraday's law. Are your graphs consistent with the minus sign in it?

6. In a loudspeaker a coil is attached to a cardboard cone, which can move in the field of a permanent magnet. Describe its production of sound.

7. Describe the action of a magnetic (or "dynamic") microphone.