

The Nucleus: Heart of the Atom

Henri Becquerel, Marie Curie, and the beginning of nuclear physics

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Particles

The existence of the atomic nucleus was discovered in 1911. The next quarter of a century saw the flowering of nuclear physics, with a gradual growth in the understanding of nuclear structure, nuclear radiations, and nuclear reactions. The composition of the nucleus was seen to determine the element to which an atom belongs. With the possibility of nuclear transformation, elements no longer needed to be considered permanent and immutable.

The first nuclear transmutations that were observed were the ones that occur spontaneously in radioactive materials. The large amount of energy released in radioactivity was a puzzle from the beginning. Eventually the large-scale release of energy in stars was also understood to be the result of nuclear reactions. But it was in 1939, with the discovery of fission, that nuclear physics left the sheltered domain of the physics laboratory. The first nuclear reactor began operation in 1942, and the first atomic (really nuclear) bombs were detonated in 1945. The genie was out of the bottle, and we are still trying to learn how to live with it.

13.1 Henri Becquerel, Marie Curie, and the beginning of nuclear physics

In 1896 Henri Becquerel wrapped a photographic plate so that no light could get to it together with a uranium compound outside the package. It was to be an experiment on x-rays, which had been discovered by Röntgen a few weeks earlier. He meant to investigate the possibility that sunlight would somehow stimulate the uranium to emit x-rays, which would then pass through the packaging and expose the plate, so that it would be darkened after being developed. The darkening did indeed take place, but did so even in the absence of any light on the uranium. Becquerel's hypothesis was that in the absence of any external energy source there would be no radiation coming from the uranium, but somehow that's what was happening. It was clear that a new and previously unsuspected process was taking place.

This discovery of the *radioactivity* of uranium can be said to mark the birth of nuclear physics. At that time, however, this was not apparent, because it was not until 15 years later that it was realized that the atom had a nucleus.

The subject of radioactivity was taken up by Marie Curie when she started the work toward her doctorate, at a time when no woman in Europe had achieved this degree. She had come to Paris from her native Poland in 1891 as Maria Skłodowska to study physics. Four years later she married Pierre Curie, who was eight years

older, a physicist who already had important and lasting accomplishments to his credit. Their first child was born in 1897, and near the end of that same year she began the work toward her doctor's thesis. Within a few weeks she showed that the uranium-containing mineral pitchblende emitted more radiation than the uranium itself. She concluded that this material must contain another substance more radioactive than the uranium. This was the fundamental discovery that led directly to her later successes.

The first report to the French Academy of Sciences carried only her name as author, and later she was quite firm in specifying that the paper was about her idea and her work. Later that year both Curies showed that there were, in fact, two such substances, namely the two elements that they named polonium and radium. Marie Curie took on the work of isolating them, a task that proved to be formidable and took four more years.

For many people it was quite difficult to accept the fact that a fundamental discovery was made by a woman, a beginning graduate student, alone, at a time when she was closely associated in her personal and professional life with a senior experienced physicist. Subsequently the names of Pierre and Marie Curie were closely linked, but the major credit was often given to him. Both of them received the Nobel Prize in physics for 1903 (jointly with Becquerel), but it eventually became known that a number of physicists had nominated only Pierre. In 1911, the same year that she received her second Nobel Prize (this

time in chemistry), Marie Curie was rejected for membership in the Academy of Sciences.

Eventually her reputation became strong and widespread. But in the popular mind she was identified much more with the four-year work of purification, boiling, precipitating, and separating ever more concentrated samples of the new elements, perhaps because that seemed closer to the accepted view of “women’s work.” It was, however, the more abstract and intellectual achievement of predicting the existence of the new substances that was her most significant contribution.

There are two further features of the initial work that were fundamental for the development of the field. One is that she understood that the radiation is independent of chemical combinations or other molecular properties, that it is, in other words, associated with the atoms themselves. (Later it would be realized that it comes from the atomic nucleus.) The other is that the radiation can be used to identify a material even when the sample is so small that chemical methods cannot be used.

During the next few years it was shown that there are three kinds of radiation that were called alpha, beta, and gamma rays. In time they were identified as helium nuclei, electrons, and high-energy photons, respectively.

The crucial discovery that most of the mass of an atom is concentrated in a nucleus at its center came in 1911. It was based on an experiment in the laboratory of Ernest Rutherford in England, in which a beam of alpha particles was directed at a gold foil. The unexpected result was that some of the alpha particles were deflected through large angles, with some even reflected back along their original direction. It was known that the atom consists of electrically-charged particles, but they were thought to be uniformly distributed throughout its volume. The electrical force on the positively-charged alpha particle would then be relatively small, too small to cause the large deflections. Later Rutherford would describe the observation by saying “It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you.” To see why he said this, it helps to remember how vast the distance between the nucleus and the electrons is compared to the nuclear size.

Rutherford was able to show that the deflection is consistent with a model of the atom in

which the positive charge and almost all the mass is concentrated in a nucleus at a point inside the atom. Collisions between the alpha particle and the gold nucleus can occur when the distance between them is very small. The electric force on the alpha particle can then, in accord with Coulomb’s law, be very large, large enough to deflect the alpha particle through the observed large angles.

The discovery of the nucleus led soon afterward to the development of Bohr’s model of the atom, but a more complete knowledge of the structure of the nucleus became possible only after the discovery of the neutron in 1932.

13.2 What is the universe made of? The stable nuclei and their binding energy

Nuclear structure

The discovery of the neutron was essential for the understanding of the structure of nuclei. Here is a review of the basic results that emerged.

Nuclei are composed of protons and neutrons, collectively called *nucleons*. The number of protons, Z , is called the *atomic number*, and it determines the position in the periodic table of elements. In a neutral atom this is also the number of electrons in the atom. (An atom in which the number of electrons is not equal to Z is electrically charged and is called an *ion*.)

The number of neutrons in a nucleus is its *neutron number*, N . A particular species of nucleus, characterized by its values of Z and N , is called a *nuclide*. For a particular value of Z , i.e., for a particular element, there may be nuclides with different values of N , each called an *isotope* of the element.

The number of nucleons in a nucleus, $Z + N$, is called its *mass number*, A , because the nucleons represent almost all of the mass of an atom. The notation for the corresponding nuclide is ${}_Z^AX$, where X is the symbol for the element whose atomic number is Z .

The nuclei and nuclides of hydrogen have special names. “Ordinary” hydrogen (${}^1\text{H}$), whose nucleus is the proton, comprises 99.985% of natural hydrogen. The rest is the isotope ${}^2\text{H}$, with $Z = 1, N = 1$, and $A = 2$. It is called *deuterium* or *heavy hydrogen*, and its nucleus is

called the *deuteron*. Water whose hydrogen is deuterium is called *heavy water*.

There is a third isotope, ^3H , *tritium*, whose nucleus consists of a proton and two neutrons. It is radioactive and is not part of natural hydrogen.

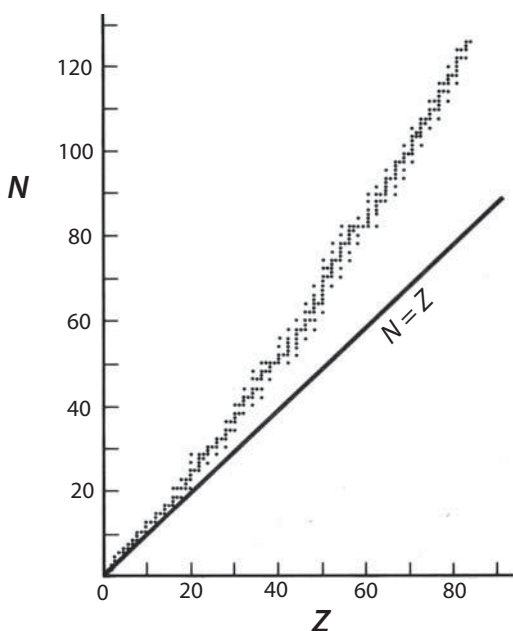
EXAMPLE 1

The atomic number of oxygen is 8. The most abundant isotope of oxygen is oxygen 16, with the symbol $^{16}_8\text{O}$, or just ^{16}O , since the fact that we are talking about oxygen automatically means that the atomic number is 8.

What is the composition of a nucleus of ^{16}O and of the other naturally occurring isotopes of oxygen, ^{17}O and ^{18}O ?

Ans.:

The atomic number of oxygen is 8, so that all isotopes of oxygen have 8 protons in the nucleus. Oxygen 16 has $A = 16$ and consists of 16 nucleons. Eight of them are protons and the rest (also eight) are neutrons. The other isotopes, ^{17}O and ^{18}O , have 9 and 10 neutrons, respectively.



Some nuclides are stable and continue to exist indefinitely. Some break apart (*decay*) spontaneously and are *radioactive*. During the decay

process they emit radiation, most commonly alpha and beta particles, and are transformed into nuclides of other elements. All of the elements with Z greater than 83, such as uranium ($Z = 92$) and radium ($Z = 88$), are radioactive and have no stable isotopes.

The figure shows all the stable nuclides on a graph of N against Z . It reveals some important characteristics of nuclei and of the forces within them.

We see that on this graph the stable nuclides are in a narrow band called the *region of stability*.

That nuclei exist at all shows that there must be a *nuclear force* that acts to attract nucleons to each other. Since nuclei contain protons, the nuclear force inside the nucleus must be stronger than the electric force. The electric force acts only between the positively charged protons, but it acts in the opposite direction, repelling them from each other. The fact that the region starts out, for small Z , along the line $Z = N$ leads us to believe that nature favors equal numbers of protons and neutrons, and that the nuclear force acts equally between neutrons and protons.

The region of stability then deviates from the line $Z = N$, with N gradually getting larger and larger than Z . We can understand this as a result of the electric force. As the number of protons in the nucleus increases, the electric repulsion between them plays a greater and greater role. Neutrons are not affected by the electric force and are therefore favored. The result is that the ratio of neutrons to protons becomes larger as the number of protons increases.

In the region of stability the nuclear force of attraction between the nucleons predominates. The electric force of repulsion between the protons, which acts to disrupt the nucleus, eventually results in the end of the region of stability at $Z = 83$.

A closer examination of the naturally occurring nuclides shows that even numbers are favored for both Z and N . There are only four nuclides with odd Z and odd N , and they are near the origin of the graph: ^2H , ^6Li , ^{10}B , and ^{14}N . On the other hand there are 49 nuclides with odd Z and even N , 51 nuclides with odd N and even Z , and 157 nuclides with even Z and even N .

Nuclear energies

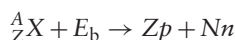
The relation $E = Mc^2$ was developed by Einstein in 1905 as a result of the special theory of relativity. Even at that time, when there was no knowledge of the existence of nuclei, he suggested that the release of energy accompanying the decay of radium was so large that it might provide a test of the mass-energy equivalence. He realized that the necessary precision for the measurement of mass was then not available. A test and verification involving nuclear energies was made, but not until 1932, shortly after the discovery of the neutron.

Since $c = 3 \times 10^8$ m/s, Einstein's relation shows that 1 kg of mass is equivalent to the enormous amount of energy 9×10^{16} J. The mass unit that is commonly used for atoms and nuclei is the atomic mass unit, u , defined as $\frac{1}{12}$ the mass of a neutral atom of ^{12}C , or 1.660×10^{-27} kg. Its energy equivalent is 931.5 MeV. The mass of the proton is 1.007 u , equivalent to 937.5 MeV.

Since mass and energy are equivalent, mass and energy units can be used interchangeably, and we can say simply that the mass of the proton is 937.5 MeV.

Binding energy

The *total binding energy* of a nucleus, E_b , is the amount of energy that would have to be supplied to it to separate it into its separate A nucleons. We can write the process as



On the left side is the original nucleus, and we add the binding energy to it. On the right side are the separated Z protons and N neutrons. We can translate the relation into an energy-balance equation:

$$m_X + E_b = Zm_p + Nm_n$$

which says that the mass of the original nucleus plus the binding energy is equal to the sum of the masses of the separated Z protons and N neutrons. In the application of this and similar equations it is, of course, necessary to use the same units for each term. They can be mass units, such as kg or u , or they can be energy units, such as J or MeV.

EXAMPLE 2

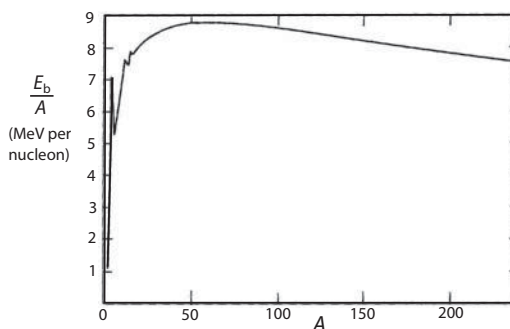
$m_p = 1.007276$ u and $m_n = 1.008665$ u . The total binding energy of ^{16}O is 127.6 MeV. What is the mass of a ^{16}O nucleus?

Ans.:

$Z = 8$ and $N = 8$. Use atomic mass units: $8m_p = 8.058206$ u . $8m_n = 8.06932$ u . The sum is 16.127526 u .

We need to subtract the binding energy. First convert it to u : $1 u = 931.49$ MeV. 127.6 MeV = .136985 u . The mass of the oxygen nucleus is $m(^{16}\text{O}) = 8m_p + 8m_n - E_b = 16.127526 - 0.136985 = 15.990535$ u .

(Tables of masses usually give the masses of the neutral atoms, which are greater by the masses of the electrons. For oxygen this means the addition of eight electron masses or 0.00439 u .)



The figure shows one of the most informative and interesting graphs in nuclear physics. On the vertical axis is the binding energy divided by the mass number, $\frac{E_b}{A}$, and on the horizontal axis is the mass number, A . It is determined by plotting a point on the graph for each stable nuclide. The line closely follows the experimental values. On this graph the higher the point for a given nuclide, the greater is its *binding energy per nucleon* and the more stable the corresponding nucleus.

We see that from about $A = 10$ on, the curve varies by only about $\frac{1}{2}$ MeV on either side of 8 MeV. In other words, the binding energy per nucleon is, to a first approximation, about constant in this region. This is very different from the binding energies of atoms, where the arrangement of electrons in shells and subshells leads to a hierarchical structure with large variations of

the binding energy. In contrast, the nucleus is seen to be quite egalitarian.

The situation is similar to that of molecules in a drop of water, where each is equivalent and each is bound equally to the others. The model of the nucleus that behaves this way is therefore called the *liquid drop model*.

It is easy to understand the downward trend on the right-hand side of the graph. As we go to the right, more and more protons are in the nucleus, and the disruptive effect of the electric force of repulsion between them makes itself felt more and more as it decreases the binding energy, i.e., the energy that needs to be added to disrupt the nucleus.

The most stable nuclides are seen to be in the vicinity of $A = 50$, so that they are near $Z = 26$, which is the atomic number of iron. At the left we see peaks for $A = 4, 12$, and 16 , i.e., for the alpha particle and two of its multiples, indicating that the alpha particle (the helium nucleus, ${}^4_2\text{He}$) is a particularly stable configuration. The nucleus with the smallest binding energy is the deuteron. It takes only about 2 MeV to separate its proton and neutron, so that $\frac{E_b}{A}$ for it is about 1 MeV.

EXAMPLE 3

Estimate how much energy it would take to decompose a nucleus with $A = 100$ into its separated nucleons.

Ans.:

The figure shows $\frac{E_b}{A}$ to be about 8.6 MeV. The total binding energy is therefore about $(8.6)(100) = 860$ MeV.

EXAMPLE 4

Think of the different ways that 200 nucleons can arrange themselves. Which combination is more stable—a single nucleus with $A = 200$ or two nuclei, each with $A = 100$?

Ans.:

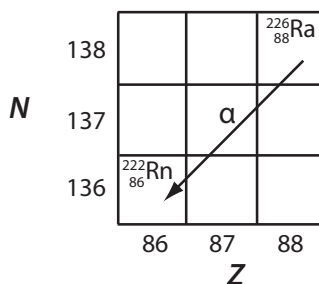
For $A = 200$ the figure shows $\frac{E_b}{A}$ to be about 7.8 MeV, for a total binding energy of about 1560 MeV. From the answer to Example 3 we see that the total binding energy for two nuclei of 100 nucleons each would be twice 860 MeV, or 1720 MeV. The larger number means a larger binding energy and a more stable nucleus.

13.3 Radioactivity

Unstable, radioactive nuclides exist in the vicinity of the region of stability. The radioactive nuclides undergo spontaneous transformations toward greater stability. They change into nuclides that have larger binding energy and are either stable, or at least closer to the region of stability. This can happen by several processes. The main ones are the emission of alpha particles, or alpha decay, and the emission of beta particles, or beta decay. The two are fundamentally different, and we will discuss them in turn.

Alpha decay

The alpha particle was shown by Rutherford and his co-workers to be the nucleus of ${}^4_2\text{He}$. A radioactive nucleus that emits an alpha particle transforms into a daughter nucleus whose atomic number is smaller by 2 as a result of the loss of two protons, and whose mass number is smaller by 4 as a result of the loss of four nucleons. On the figure of N against Z , where each box represents a nuclide, this corresponds to a move two steps down and two to the left, i.e., a move along the line $N = Z$ at 45° to the two axes.



Examples of alpha-emitting nuclides are ${}^{238}_{92}\text{U}$, ${}^{235}_{92}\text{U}$, and ${}^{226}_{88}\text{Ra}$. The process can be written in the form of a relation such as ${}^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + \alpha$. The amount of energy that is transformed into kinetic energy may be included:



In this form it represents an equation that describes the mass and energy transformations quantitatively. On the left is the mass of the parent nucleus. On the right are the masses of the daughter nucleus and the alpha particle as well as the released kinetic energy:

$$M_{\text{Ra}} = M_{\text{Rn}} + M_{\alpha} + 4.76 \text{ MeV}$$

where the masses are those of the nuclei of the isotopes ^{226}Ra and ^{222}Rn .

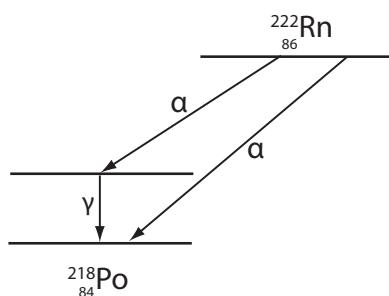
In any calculation the units of each term must, of course, be the same. (Usually atomic mass units are used.) If all the masses are known, the released energy can be calculated. In other cases the released energy may be measured, and an unknown mass calculated.

EXAMPLE 5

The nucleus of ^{226}Ra is heavier than the nucleus of ^{222}Rn . What is the mass difference, in kg, in atomic mass units and in MeV?

Ans.:

We can turn the relation between the masses around to say that $M_{\text{Ra}} - M_{\text{Rn}} = M_{\alpha} + 4.76 \text{ MeV}$. The difference is the mass of the alpha particle plus the binding energy of 4.76 MeV. The mass of the alpha particle is 4.00150 u. The extra binding energy, in atomic mass units, is $\frac{4.76}{931.5} = 0.00511 \text{ u}$. The difference is therefore 4.00661 u or 3732 MeV, or $6.653 \times 10^{-27} \text{ kg}$.



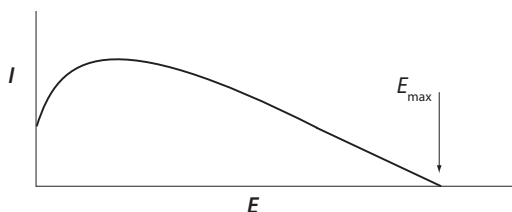
The emission of the alpha particle is a kind of explosion. Energy is released as a result of the transformation of internal energy to kinetic energy. Momentum is conserved, and the daughter nucleus recoils with a momentum equal in magnitude and opposite in direction to that of the emitted alpha particle. In our example the mass

of the nucleus is about 56 times that of the alpha particle, so that its velocity and kinetic energy are correspondingly smaller, and are often neglected.

If the daughter nucleus is in its ground state, all of the released energy becomes kinetic energy. The nucleus may also be left in an excited state. In that case the excited state subsequently goes to the ground state with the emission of a photon. A photon emitted by a nucleus is called a *gamma ray*. The figure shows an example of an energy-level diagram or *decay scheme* for the alpha decay of ^{222}Rn . If instead of going directly to the ground state, the daughter nucleus is left in the excited state, a gamma ray with an energy of 0.51 MeV accompanies the alpha decay.

Beta decay

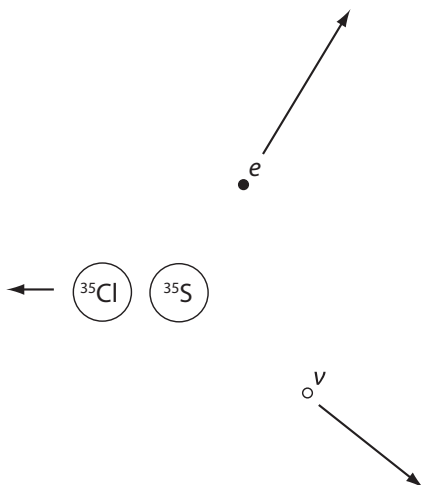
Beta particles were identified as electrons in 1902, even before the identity of the alphas was established in 1908. The alpha particles were soon seen to have definite energies, equal to the difference in the energies of the parent and daughter nuclei. It was confidently expected that this would also be true for the betas. It was therefore a major surprise when it was gradually realized that this was not so.



It was found that the emitted electrons have a continuous range of energies, from very low to a definite maximum. And it is that maximum energy that is equal to the difference in the energies of the parent and daughter nuclei.

The problem was that in spite of diligent efforts, no other particles were observed. It seemed therefore that the beta decay process violated those cornerstones of science, the law of conservation of energy and the law of conservation of momentum. Wolfgang Pauli, in 1930, made the hypothesis that there is another particle that carries away the missing energy, even though none had been observed. That particle (now called the *neutrino*) would have to be able

to traverse large amounts of material without interacting with it as it passes through.

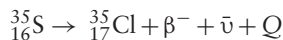


The neutrino hypothesis allowed the preservation of the laws of conservation of energy and momentum. For the next quarter of a century all efforts to gain direct evidence for neutrinos failed. Nevertheless they seemed so necessary and accounted for so many details of the beta-decay process that their existence came to be accepted. Eventually, in 1956, neutrinos were observed directly.

In the meantime, in 1932, a new particle, the *positron*, was discovered. It has the same mass and most other properties as the electron, but it is positively charged. Two years later it was shown (by the Curie's daughter Irène and her husband Frederic Joliot) that it can also be the result of beta decay.

Positive beta decay is also accompanied by a neutrino. Electrons and positrons are *antiparticles* of one another. (We saw in Chapter 12 that their meeting leads to annihilation.) Similarly the neutrinos emitted in positive and negative beta decay are antiparticles of one another. The one accompanying β^- decay is called the *antineutrino*, $\bar{\nu}$ ("nu-bar," where the bar indicates an antiparticle), and the one accompanying β^+ decay is called the neutrino, ν . Generically, both are usually just called neutrinos. Much later other kinds of neutrinos were discovered, and when it is necessary to distinguish those that accompany beta decay they are called *electron neutrinos*.

An example of a relation representing β^- decay is



Here the symbol Q represents the released energy that is shared as kinetic energy by the three decay products, namely the electron, the antineutrino, and the residual nucleus. Because of its greater mass, the kinetic energy of the recoiling nucleus is even smaller than for alpha decay and it is shared almost entirely by the electron and the neutrino. Sometimes the electron gets all the energy, sometimes the neutrino gets it all, and most of the time each gets part of it. The energy of the electron can therefore be any amount between zero and the maximum energy, Q .

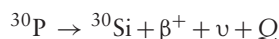
EXAMPLE 6

The mass of the nucleus of ${}^{35}\text{S}$ is heavier than that of ${}^{35}\text{Cl}$ by 0.00073 u. What can you say about the energies of the emitted beta rays?

Ans.:

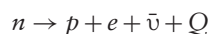
When a beta particle is emitted by ${}^{35}\text{S}$, some of the mass that the sulfur nucleus loses is that of the emitted electron (0.000549 u). Most of the rest is that of the ${}^{35}\text{Cl}$ "daughter" nucleus. The remaining mass is shared by the electron and the neutrino as kinetic energy. Here it is $.00073 - .00055 = .00018$ u, which is equivalent to 0.17 MeV. The kinetic energy of the beta particle can be anywhere between zero and 0.17 MeV.

An example of β^+ decay (the one found by the Joliot-Curies) is



In alpha decay the emitted particle is a constituent of the parent nucleus. But there are no electrons in the nucleus—they are created at the time of the disintegration. In β^- decay one of the neutrons in the nucleus changes into a proton, and an electron and an antineutrino are emitted.

The neutron decays into a proton even when it is free, outside a nucleus, in accord with the relation



with the energy balance

$$m_n = m_p + m_e + m_{\bar{\nu}} + Q$$

The neutrino mass, m_ν , is much smaller than that of the electron. For many years it was thought to be zero, but more recent experiments indicate that it has a very small mass. We can neglect it most of the time. The known masses of the neutron, the proton, and the electron show Q for this case to be 0.78 MeV.

EXAMPLE 7

Can the proton decay into a neutron? Explain.

Ans.:

The transformation of a proton into a neutron, together with a positron and a neutrino, cannot occur for a free proton, since the proton's mass is smaller than that of the neutron. There is no internal energy that can be given away and still leave a positive value for Q .

If, on the other hand, the proton is part of a nucleus, the internal energy is shared by all the nucleons, and a radioactive decay can take place in which the proton changes to a neutron. This is what happens in β^+ decay. Of course it can occur only if the mass of the initial nucleus is larger than the sum of the masses of the daughter nucleus and one electron.

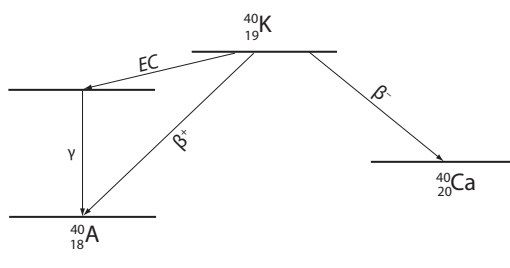
There is an alternative to β^+ decay that also leads to a daughter nuclide with Z less by one and N greater by one: instead of emitting a positron a nucleus may absorb one of the atom's orbiting electrons. In this process, called *electron capture*, the positron does not need to be created, so that less energy is required. On the other hand, since the atomic electrons spend most of their time far from the nucleus, the probability of electron capture is usually much less than that of positron decay when both are possible. Electron capture is primarily important when there is enough energy for it to occur, but not enough for positron decay.

What is the difference in the energy that is required for the two processes to take place? Write down the relation for positron decay as $P \rightarrow D + \beta^+ + \nu + Q_1$, where P is the parent nucleus, D is the daughter nucleus, and Q_1 is the energy released in the process. The corresponding mass relation is $M_P = M_D + m_e + Q_1$, where M_P and M_D are the nuclear masses of the parent and daughter and m_e is the electronic mass, so that $Q_1 = M_P - M_D - m_e$.

For electron capture the relations are $P + e \rightarrow D + \nu + Q_2$ and $M_P + m_e = M_D + Q_2$ or

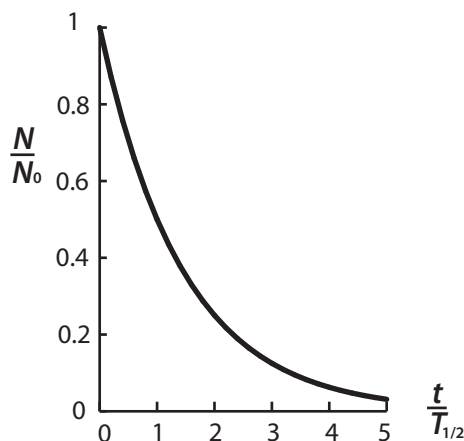
$Q_2 = M_P - M_D + m_e$. We see that Q_2 is larger than Q_1 by $2m_e$! In other words, electron capture releases more energy than positron decay by 1.02 MeV. If the difference in energy between the parent's state and the daughter's state is positive, but less than 1.02 MeV, electron capture can take place, but positron decay cannot.

^{40}K is a nuclide that can decay by β^- and β^+ emission and by electron capture. It is 0.12% of naturally occurring potassium.



Half-life

Soon after the discovery of radioactivity it was found that the amount of radiation decreases with time, and that the change can be described by a *half-life*, $T_{1/2}$. After one half-life the rate at which particles are emitted decreases to one-half of the original value, after a second half-life it goes to 25%, and so on. The rate of particle emission is proportional to the amount of radioactive material. Hence the original amount of material decreases to $\frac{1}{2}$ after one half-life, to $\frac{1}{4}$ after two half-lives, to $\frac{1}{8}$ of the original amount after three half-lives, and so on. In general, after n half-lives



the amount of material as well as the rate of emitted particles decrease to 2^{-n} times their original value. The graph shows $\frac{N}{N_0}$ plotted as a function of time, as $\frac{t}{T_{1/2}}$. $\frac{N}{N_0}$ is the fraction of material remaining and $\frac{t}{T_{1/2}}$ is the time measured as a multiple of the half-life.

Intermediate values can be calculated from the *decay equation*

$$N = N_0 e^{-\lambda t}$$

where N is the number of nuclei at time t and N_0 is the number at time $t = 0$. The exponent λ (called the decay constant) describes how quickly the number of nuclei decreases.

[We can also write this expression differently by using natural logarithms: if $e^x = A$, then the exponent, x , to which e has to be raised to give A , is the natural logarithm ($\ln$) of A . Here $\frac{N}{N_0} = e^{-\lambda t}$, so that $\ln \frac{N}{N_0} = -\lambda t$ or $\ln \frac{N_0}{N} = \lambda t$].

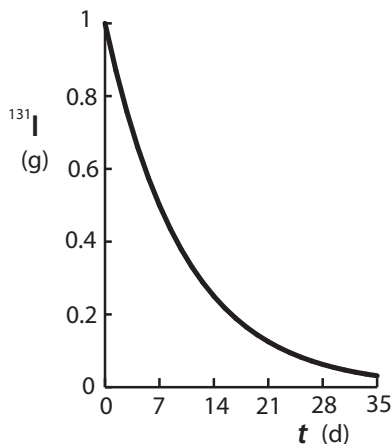
After one half-life $t = T_{1/2}$ and $N = \frac{N_0}{2}$. We can substitute these values in the decay equation to get $e^{\lambda T_{1/2}} = 2$, which can also be written as $\lambda T_{1/2} = \ln 2 = 0.693$, so that $\lambda = \frac{0.693}{T_{1/2}}$.

The number of disintegrations per second is called the *activity*. The SI unit for activity is the *becquerel* (Bq), equal to one disintegration per second. An older unit that is still widely used is the *Curie*, Ci, equal to 3.7×10^{10} Bq. It was originally defined as the activity of one gram of radium.

The activity is the rate at which the number of nuclei (N) changes. It is the slope on a graph of N against t , which we can write as $\frac{dN}{dt}$. If $N = N_0 e^{-\lambda t}$, then $\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$, which is equal to $-\lambda N$. The relation $\frac{dN}{dt} = -\lambda N$ shows that the activity ($\frac{dN}{dt}$) is proportional to the amount of material (N).

EXAMPLE 8

- Draw a graph of the decay of ^{131}I , plotting the mass of the radioactive material as a function of time. At $t = 0$ there is 1 g of the iodine. Its half-life is 7 days.
- How much iodine is left after 2.5 half-lives?
- How long does it take for the activity to decrease to 10% of the original amount?



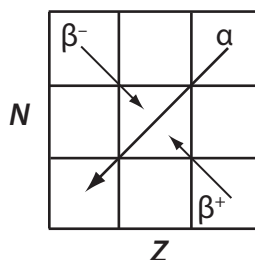
- Check your answers to parts (b) and (c) by looking at the graph in part (a).

Ans.:

- $N = N_0 e^{-\lambda t} = N_0 e^{-\frac{0.693}{T_{1/2}} t} = N_0 e^{-(0.693)(2.5)} = N_0 e^{0.173} = 0.177 N_0$. After $2\frac{1}{2}$ half-lives (or 17.5 days) 0.177 of the original amount or 0.177 g is left.
- For $\frac{N}{N_0} = 0.1$, $e^{-\frac{0.693}{T_{1/2}} t} = 0.1$, or $\ln 0.1 = -\frac{0.693}{T_{1/2}} t$, i.e., $-\frac{0.693}{T_{1/2}} t = -2.30$, or $\frac{t}{T_{1/2}} = 3.32$: it takes 3.32 half-lives or 23.2 days for the original amount to decay to 10% of its original value.

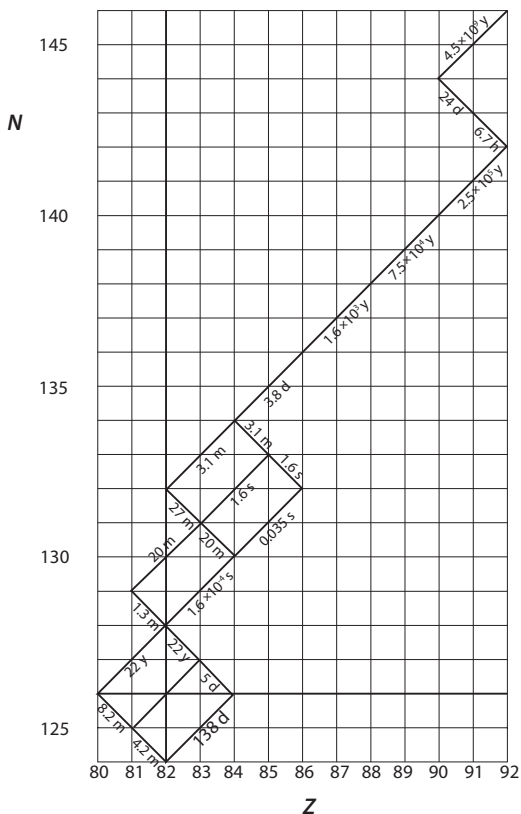
Stability and decay series

In both kinds of beta decay the parent and daughter nuclides have the same mass number, A . β^- decay is represented by an increase in Z of one and a decrease in N of one, i.e., one step down and one step to the right. It allows a neutron-rich nuclide to get closer to the region of stability from



the left. Similarly β^+ decay occurs for nuclides to the right on the diagram so that they can get closer to the region of stability.

Alpha-particle decay occurs primarily for the nuclides at the upper end of the periodic table of elements, beyond the last stable one (^{83}Bi). In this region the nuclides form *decay series*, such as the one that starts with ^{238}U and proceeds through a number of alpha and beta decays until it reaches the stable nuclide ^{206}Pb , as shown on the diagram. One nuclide in the chain is ^{226}Ra with its half-life of 1600 years, followed by ^{222}Rn , whose half-life is 3.8 days. It leads to other alpha and beta emitters, including ^{210}Bi , which has a half-life of 22 years.



himself to radioactive sources purposely to study the effect. The damage is no surprise since even ultraviolet radiation, with its much less energetic photons, can cause sunburn and, as we know today, cancer.

The possibility of delayed effects was not, however, realized, and insufficient precautions were taken. Marie Curie lived until she was 67, but it seems clear that her health suffered from the effects of the radiation with which she worked. In the biography of her mother, her daughter Eve went so far as to quote what she calls the verdict of science: "Marie Curie can be counted among the eventual victims of the radioactive bodies which she and her husband discovered."

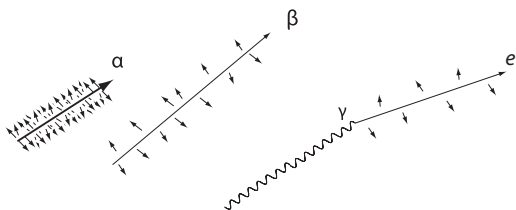
By the 1920s it had become evident that radioactive radiations could cause severe damage and death. The best-known case, which served to publicize the dangers, was that of the women who were employed at the U.S. Radium Corporation in New Jersey to paint watch dials with luminous paint containing radium. They pointed their brushes with their lips, and so absorbed the radioactive material, leading to bone destruction, bone cancer, blood disorders, and eventually 41 deaths among them. (Radium is in the same column of the periodic table of elements as calcium, and like it, once in the body it travels preferentially to the bones.)

At the same time various curative powers were ascribed to radium, and health tonics and medicines containing it were used until the 1950s.

Ionization and excitation

Since the particles emitted by radioactive materials have energies up to several MeV, they can cause the disruption of atoms and molecules that they encounter. The main effect is that charged particles can remove electrons from atoms just by passing close to them, as a result of the Coulomb force. In other words they can cause ionization of the atoms. In other cases they can give the atoms that they pass the energy to go to higher energy levels, from which they then return to the ground state. (This second effect was used in the luminous paints mentioned in the previous section. The radium was mixed with zinc sulfide, which is excited to higher energy levels by the alpha particles from the radium, and then

emits visible radiation on its return to the ground state.)



Charged particles, such as alpha and beta particles, can give rise to ionization and excitation directly. Gamma rays and neutrons don't, but they can do so indirectly by first giving some of their energy to charged particles. Gamma rays can give energy to electrons by the photoelectric effect, the Compton effect, and by pair production. Neutrons can give up energy by collisions with nuclei and through nuclear reactions.

Units

A unit that is used to measure the amount of ionization is the *roentgen* (R) (or röntgen). One roentgen is the amount that produces 2.58×10^{-4} coulombs of electrons per kg of air at standard temperature and pressure (STP), i.e., at 0°C and one atmosphere of pressure. (This amount of charge is an obsolete unit of charge called the *statcoulomb*.)

The amount of ionization that is produced is different for different materials. In air at STP it takes, on average, 34 eV of absorbed energy to produce one ion pair.

The roentgen measures the amount of radiation by measuring the ionization that it produces in a standard sample of air. Other units are needed to measure the effect of the radiation. The most important quantity that characterizes the effect of the radiation is the energy that is absorbed per unit mass. This is called the *absorbed dose*, and the SI unit for it is the *gray* (Gy), equal to 1 J/kg . An older unit is the *rad*, equal to 10^{-2} Gy .

Ionizing radiation causes biological damage principally by disrupting molecules that it passes. The absorbed energy per unit mass is a good guide for beta and gamma radiation, but alpha particles and neutrons cause more damage for the same absorbed dose. This has led to the introduction of an empirically determined quantity called the *quality factor* (QF) (also called the *relative*

biological effect, RBE) to take into account the fact that some types of radiations cause more damage than others. It is equal to one for betas and gammas, to 20 for alphas, to 3 for slow neutrons, and to 10 for fast neutrons. When the absorbed dose (in Gy) is multiplied by the QF it becomes the *effective dose*, for which the SI unit is the *sievert* (Sv). (If the absorbed dose is measured in *rads* and is then multiplied by the QF, it becomes the effective dose in *rems*.)

The QF is larger for alphas because they lose their energy in a shorter distance, so that the ionization is more densely distributed. A 1 MeV alpha particle travels about 0.56 cm in air before it stops, and about 0.1% of that distance in tissue. Electrons with the same energy travel about 3.1 m in air.

Gamma rays do not travel a definite distance (or *range*) and then stop. Their intensity decreases exponentially. A 1 MeV gamma ray beam in air will be attenuated by half in about 75 m of air and in 0.85 cm of lead.

We see that an alpha source does not pose a threat as long as it is outside the body and at least a few cm from it. Once ingested, however, an alpha emitter can cause major damage. One of the most important examples is radon (^{222}Rn), which emits 5.5 MeV alpha particles as it decays to ^{218}Po with a half-life of 3.8 days. It is a decay product of uranium 238 that (in small quantities) is widely distributed in rocks and hence in building materials such as bricks. Radon is a gas, and if it is breathed in and decays in the lungs it leaves its daughter products, which emit further alpha, beta, and gamma rays. The alphas, in particular, because of their short range, cause dense ionization and hence the greatest damage.

Radiation to the whole body of 0.25 to 1 Sv causes changes, primarily in the blood, that the affected individual may not be aware of. "Radiation sickness" with malaise and vomiting will result from 1 to 3 Sv, but the person is likely to recover. Higher doses lead to widespread damage throughout the body, particularly to the gastrointestinal tract and the bone marrow. With a dose of 5 Sv the chance of recovery is about 50%. Above 6 Sv death is almost certain.

The study of cancer induced by radiation is made difficult by the fact that cancer is a common disease that is usually caused by other factors. Only massive doses lead to numbers that

can be distinguished from those that would occur in their absence. Some of the most important evidence comes from survivors of the atomic bombs in Hiroshima and Nagasaki. This population has been studied over the years and the doses to which people were subjected have been estimated from knowledge of where they were at the time of the exposure.

Estimates based on these and other studies lead to cancer deaths attributed to radiation of the order of 5% per sievert. This means that in a population of 100 people exposed to 1 Sv there would be five excess cancer deaths. It must be stressed that this is a rough estimate. Needless to say, it depends on many factors, some dependent on the nature of the radiation and some on the irradiated individual. Furthermore the estimate applies to survivors of high doses, since it has not been possible to demonstrate directly that radiation below 0.5 Sv can produce cancer. Its use at lower doses is highly controversial. It has been called the *LNT hypothesis*, standing for “linear—no threshold.” It implies that there is no minimum dose for radiation-induced cancer, and that the variation of the cancer incidence with dose (the *dose-response relationship*) remains linear to the lowest doses. The result would be that even the amount of background radiation to which everyone is exposed would lead to some development of cancer. Some groups recommend the use of the LNT hypothesis because it represents a conservative approach in the face of unknowns. Others believe that its conclusion that all radiation carries risks, regardless of dose, is unrealistic, unsupported by the data, and leads to large expenditures of public funds that are used to reduce amounts of radiation that have not been shown to be harmful.

Background radiation

The average natural background radiation in the United States is estimated to be about 3 mSv/year. Of this amount about 60% comes from radon, and the rest, in very roughly equal amounts, from cosmic radiation, radioactive materials in the earth, and radioactive material inside the body, principally ^{14}C and ^{40}K . Another 0.5 mSv is from man-made sources, principally medical x-rays and nuclear radiations. These numbers are obviously quite variable, depending on location and other factors.

13.5 The nuclear force

Before the discovery of the neutron, nuclei were thought to consist of protons and electrons. By 1921, however, it had become clear that electric forces were not strong enough to hold such a nucleus together. The eventual realization that nuclei consist of protons and neutrons made it definite that another kind of force is required. The only forces that were then known were the gravitational force, which is far too weak, and the electrical force, which causes the protons to repel each other and does not affect the neutrons.

At the time of his discovery of the nucleus, Rutherford showed that the scattering of alpha particles could be described by assuming only electrical forces between the alphas and a point nucleus. With this hypothesis he developed the *Rutherford scattering formula*, which was in very good agreement with the experiments. Further experiments during the following decade showed, however, that when the alphas have sufficient energy to come very close to the target nucleus, the formula no longer holds. A new force, the strong nuclear force, comes into play.

There is no simple formula for the nuclear force, similar to Newton’s law of gravitation or Coulomb’s law. Instead its primary characteristics are that it acts between nucleons regardless of whether they are protons or neutrons, that its range is so short that in a nucleus it acts only between neighboring nucleons, and that it is stronger than any other force.

The liquid drop model and beyond

We have already seen that the leading feature of the liquid drop model of the nucleus is that the binding energy depends only on the number of nucleons, so that the binding energy per nucleon is roughly constant.

As in a sphere composed of closely packed marbles, the number of nucleons is proportional to the volume of the nucleus, and we can call this part of the binding energy the *volume energy*. It is proportional to the number of nucleons, A , and hence to the nuclear volume, i.e., to the cube of the nuclear radius. Experiments show that the nuclear radius is about $1.2 \times 10^{-15} A^{\frac{1}{3}} \text{ m}$.

In the interior of the nucleus the nucleons experience forces from all sides around them. At

the surface, however, there are nucleons only on one side. At the nuclear surface the nucleons are therefore less strongly bound than in the interior. We can describe this feature by adding a negative term to the binding energy. It is proportional to the surface of the nucleus, i.e., to the square of the nuclear radius, so that it is proportional to $A^{2/3}$.

These two terms, the volume energy and the surface energy, act in opposite directions. The volume energy is the main term that describes the attraction between the nucleons. The surface energy reduces this overall attraction.

To describe the binding energy curve further we can now add a term that takes into account the electric repulsion between the protons. This term is proportional to the square of the number of protons, Z , and in accord with Coulomb's law it is proportional to $\frac{1}{r}$, or to $A^{-1/3}$.

These three terms, the volume energy, the surface energy, and the Coulomb energy, can describe the binding energy curve quite closely. The relation that includes them is the core of what is usually called the *semiempirical binding energy relation*, first developed by C. F. von Weizsäcker.

The Semiempirical Binding Energy Relation

We can incorporate two further features in the formula. One is the tendency toward equal numbers of protons and neutrons. This "asymmetry term" is proportional to $(N - Z)^2$. It is zero when $N = Z$ and becomes larger with the difference between N and Z . In addition, this term turns out to be proportional to $\frac{1}{A}$.

Finally we can include the "odd-even" effect by including a term that is positive when both Z and N are even, negative when they are odd, and zero when one is even and the other odd. This term turns out to vary as $\frac{1}{\sqrt{A}}$.

The complete expression is

$$E_b = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(N - Z)^2}{A} \\ (\pm a_5 A^{-1/2} \text{ or zero})$$

With empirically determined coefficients this formula follows the observed binding energies to about $1\frac{1}{2}\%$ for nuclei with A greater than 20.

Both the tendency toward $N = Z$ and the odd-even effect are consequences of the Pauli exclusion principle. The proton and the neutron can be considered to be different states of the nucleon. Each has a spin angular momentum equal to that

of the electron, with two possible spin states. In an alpha particle each of these four possible states is represented.

This is the origin of the extraordinary stability of the alpha particle and its high binding energy compared to that of its neighbors, as seen by the peak at $A = 4$ on the binding energy curve. In accordance with the Pauli principle a fifth nucleon would have to be in a quite different state, with a higher energy, and hence a lower binding energy. It turns out, in fact, that there is no stable configuration with $A = 5$. As we shall see, this fact has an enormous influence on the evolution and the composition of the universe.

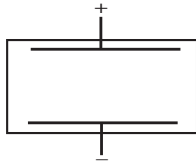
In atoms each electron moves in the electric field of the nucleus and the other electrons. The electrons are in widely spaced states determined by the Pauli exclusion principle. There is a hierarchy of states with their different quantum numbers. Nuclei are very different. As in a liquid drop their components (the nucleons) form a much more egalitarian structure. Nucleons are close to each other, but not so closely packed that they cannot move. One consequence is that there is a shell structure in nuclei somewhat like the one in atoms. In atoms the shell structure plays a dominant role. Its effect on nuclear structure is much smaller. It does lead to the fact that there are other numbers of nucleons (in addition to the number four for the alpha particle) that are relatively stable compared to the average behavior. For some time their origin was not understood and they were called "magic numbers." Eventually they were shown to be the consequence of a shell structure within the nucleus. Although the shell structure does not dominate, as it does in atoms, it still plays a significant role, sufficient to cause the *magic numbers* 2, 8, 20, 28, 50, 82, and 126 for Z and N to be favored.

13.6 Observing radioactive radiations

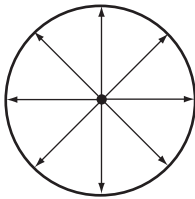
The ionization chamber and the Geiger counter

How can we detect and measure radioactivity? The first and most direct method is to measure the charge liberated by the ionization that is produced. The particles enter an *ionization chamber* containing two plates with an electric field between them. The ions that are produced

move toward the plates, constituting a current that is then measured.



A related device was developed by one of Rutherford's associates, Hans Geiger, and is known as a *Geiger counter*. It is similar to the ionization chamber in that it has two electrodes with an electric field between them. The positive electrode is a wire along the center of a glass or metal tube and the negative electrode is a metallic cylinder surrounding the wire. The important difference is that the electric field at the positive wire is much higher than that at the negative cylinder. You can see this on the figure that shows



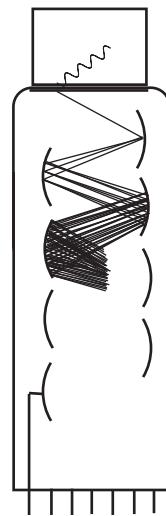
the electric field lines. They are much closer to each other at the wire in the center. An electron in the space between the electrodes is accelerated toward the wire. As it approaches the center the electron reaches an electric field that is so strong that it gives the electron sufficient energy to cause further ionization. The resulting electrons are also accelerated toward the wire and give rise to even more ionization. An avalanche of electrons arrives at the wire, triggered by just one initial ionization. The advantage is that just one initiating particle produces a pulse that is easily detected. The disadvantage is that each avalanche is the same, so that the counter gives no information about the energy of the original particle.

Scintillations

A very different method depends on the excitation to higher energy levels that is produced by the radiation. The material of the counter is chosen so as to have energy levels from which it

returns to the ground state with the emission of photons in the visible part of the spectrum. This method was already used by the Curies to detect alpha particles. The source was placed close to a zinc sulfide detector, and the resulting flashes of light, or *scintillations*, were observed under a microscope in a darkened room.

Today's scintillation counters use the same principle, but they are much more sophisticated. The detector is a transparent crystal, and instead of the tedious observations of the light flashes, they are "seen" by a sensitive phototube called a photomultiplier.



The figure shows the crystal and the photomultiplier. A gamma ray that enters the crystal gives some or all of its energy to an electron. The electron slows down in the crystal and gives up some of its energy by excitation, i.e., by causing the crystal to go to higher energy levels. The crystal goes back to the lower energy levels and emits a visible light flash as it does so. (On the figure you see one of the photons of this light flash moving through the crystal to the photomultiplier.)

At the top of the tube, inside the glass, is a thin metal coating that serves as the photo-sensitive surface. The light that falls on it gives rise to electrons through the photoelectric effect. These electrons are accelerated toward another electrode that is at a higher potential by about 100 V. That gives them the energy to knock out several electrons when they hit. Each of them is then accelerated to a further electrode, again

knocking out more electrons. After about 10 similar multiplications a single electron from the photosensitive surface produces about a million electrons at the last stage of multiplication.

A significant feature of the process is that the magnitude of the final pulse that is produced is proportional to the energy of the light flash that falls on the photosensitive surface. The energy of the light flash, in turn, is proportional to the amount of energy that is lost in the crystal by the radiation that is absorbed there.

When the combination of the crystal and the photomultiplier is used to measure the energy of the incoming radiation, it is called a *scintillation spectrometer*. It can be used to detect and measure the energy of gamma rays.

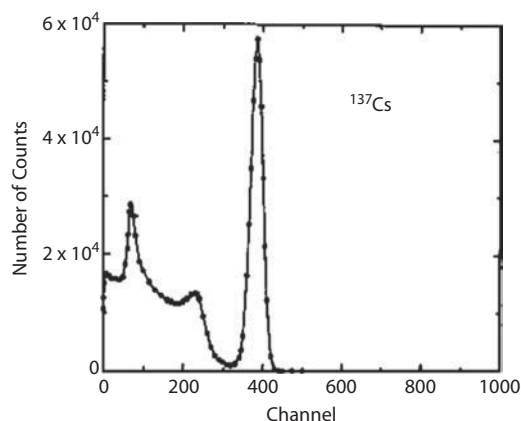
Because the crystal and photomultiplier are in a lightproof enclosure they “see” only the light produced in the crystal. The crystal is commonly sodium iodide, which is similar to sodium chloride (table salt). The higher atomic number of the iodine leads to a higher probability that inside the crystal the gamma ray is stopped by the photoelectric effect rather than by the Compton effect. (If the energy of the gamma ray is greater than 1.02 MeV there can also be pair production.)

Let’s look at what happens when gamma rays with a single energy (say 1 MeV) enter the crystal and are absorbed there. Some will be absorbed by the photoelectric effect. The resulting photoelectron has all of the energy of the gamma ray except for the electron’s binding energy of a few eV.

Do not confuse the photoelectric effect caused in the crystal by the gamma ray with the photoelectric effect caused by the visible light at the photosensitive surface of the photomultiplier. The first produces photoelectrons with the energy of the gamma ray (here 1 MeV) in the crystal. The second gives rise to photoelectrons with the energy of visible light, of the order of 1 eV, inside the photomultiplier.

Each gamma ray that is stopped in the crystal by the photoelectric effect gives rise to a light flash in the crystal of a single size. This leads, at the end of the multiplying process in the photomultiplier, to voltage pulses, all with the same size. That would make the measurement of the energy spectrum of the gamma rays straightforward. Some of the time, however, the gamma rays give up their energy through the Compton

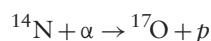
effect. In that case only part of the energy of the gamma ray is given to an electron. As a result a gamma ray with a single energy gives rise to electrons with a range of energies in the crystal. The light flashes produced in the crystal are therefore not all the same size, and neither are the voltage pulses at the output of the photomultiplier.



The figure shows the distribution of pulses initiated by a ^{137}Cs source that emits gamma rays with the single energy of 0.66 MeV. The number of the largest pulses is plotted on the right, with successively smaller pulses further to the left. The peak on the right is the “photopeak,” resulting from the electrons that get their energy through the photoelectric effect. This is the full gamma-ray energy (except for the negligibly small binding energy of the electrons). The electrons that get their energy through the Compton effect have less energy and therefore are plotted further to the left. (There is more on scintillation spectra in the problems at the end of the chapter.)

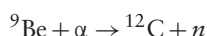
13.7 Nuclear reactions

We have already discussed the spontaneous nuclear reactions of radioactivity. In 1919 Rutherford was the first to observe a nuclear reaction initiated by bombardment with another particle. He used alpha particles emitted in radioactivity to bombard nitrogen. The reaction was



Note that the number of nucleons (18) and the number of electronic charges (9) are the same before and after the reaction. Both quantities are *conserved*.

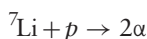
Alpha particles from polonium were also used in the discovery of the neutron. There had been speculation about the existence of such a particle for more than a decade, as well as unsuccessful experiments. In some of them neutrons were in fact observed, but wrongly thought to be high-energy photons. Finally, in 1932, following a flurry of activity in several countries, the reaction



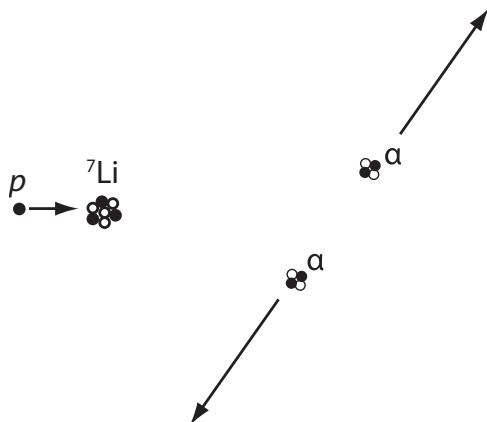
was correctly identified by Rutherford's former student James Chadwick in England.

A neutron source could then be constructed by mixing an α emitter, such as radium or polonium, with beryllium. The half-life of the neutron was later determined to be about 12 minutes.

By 1932 several devices had been built to accelerate electrically charged particles. The best known is the cyclotron, invented by Ernest Lawrence in Berkeley, California. The first reaction that was reported was



first in England, by Cockroft and Walton, using a device that they called a "voltage doubler," which produced 0.7-MeV protons, and later in the same year with 1.22-MeV protons in Lawrence's cyclotron.

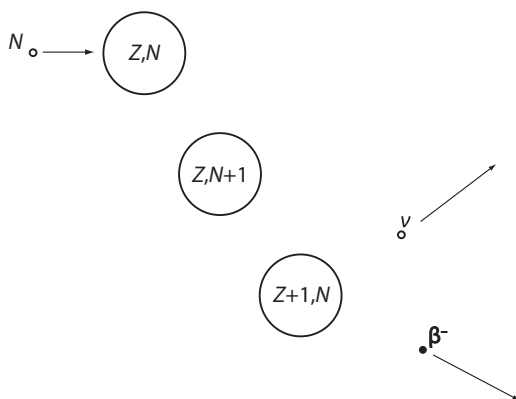


The charged particles, both the ones emitted in radioactivity and those that were given high kinetic energies in cyclotrons and other *particle accelerators*, were able to initiate nuclear reactions. To do so, however, they had to have enough energy to overcome the electric repulsion of the target nucleus, the so-called *Coulomb barrier*. Neutrons are not affected by the electric force and can come close to the nucleus even when they are moving slowly.

Neutron reactions and fission

The first person to exploit this feature in a series of experiments in which targets made of many different elements were bombarded by neutrons was Enrico Fermi in Rome.

The typical behavior that he and his collaborators found was that the neutron was absorbed by the target nucleus. In many cases the new nucleus then had too many neutrons to be stable, in other words, it was to the left and above the stability region. It would then be radioactive and emit a β^- particle to return to the stability region. The emission of the electron would, as usual, cause the daughter nucleus to have one more proton, and it would therefore be one step higher in the periodic table of elements than the initial target nucleus.



Fermi followed this behavior in a number of elements. It didn't take very long for him to ask what would happen if he went to the top of the periodic table. In 1934, soon after the start of his work with neutrons, he bombarded uranium, the naturally occurring element with the highest atomic number.

As in the other cases, he tried to identify the resulting material by studying its radioactive radiations and believed that he had discovered new *transuranic* elements with atomic numbers higher than uranium. He gave the supposed new elements names and tried to unravel the observed decay schemes, but this proved to be a difficult task because of the jumble of the resulting radiations.

Looked at from the vantage point of our present knowledge, it is strange and surprising that the mystery of what happens when uranium is bombarded with neutrons was not resolved until five more years had passed. It was 1939 when Otto Hahn and Fritz Strassmann in Germany showed by chemical means that one of the products of the bombardment seemed to be barium, element No. 56. They were hesitant to describe their result because in all nuclear reactions up to that time the changes had been no greater than that of an alpha particle, i.e., two steps in the periodic table of elements. Here was a change in the atomic number by 36!

It is clear on the binding energy curve that the nuclides near the middle of the horizontal axis have a higher binding energy per nucleon than those at the far right, and therefore represent more stable assemblies of the nucleons. However, all previous experience had been with nuclear reactions that resulted in the emission of a single particle, a proton, a neutron, an alpha particle, or no particle at all, as in the absorption of neutrons discussed earlier in this section.

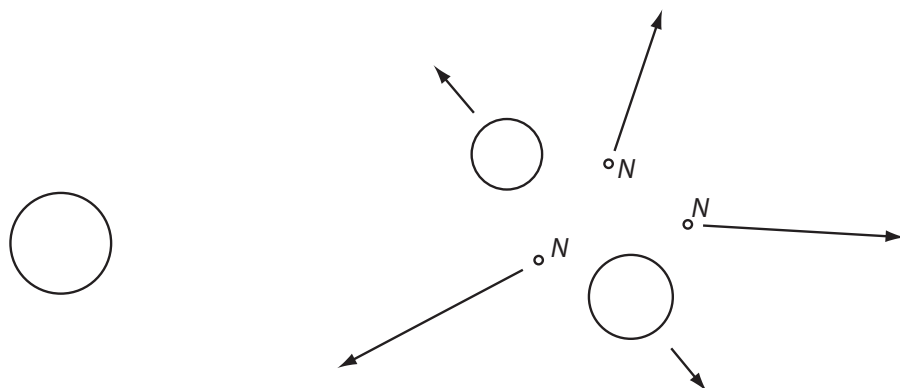
The possibility of a split of the nucleus down the middle was so different that no one considered it until Hahn and Strassmann produced the

direct chemical evidence. Even then they offered no explanation of how the barium might have been produced. It was Lise Meitner and Otto Frisch who within a short time described the process in detail in a paper that for the first time used the word “fission.”

Lise Meitner had been working closely with Hahn in Berlin since they met in 1907, she as a physicist and he as a chemist. In 1938 she left Germany abruptly, forced out because of her Jewish background. Their collaboration continued by letter as well as it could. The experiments that constitute the discovery of fission came to fruition a few months after she left, but she is widely considered to have had a major share in their success. Nevertheless, the 1944 Nobel Prize in chemistry was awarded to Hahn alone.

The binding-energy curve lets us estimate the energy that is released in the fission process. The uranium is at the far right, with values of A of 235 and 238, and has a binding energy per nucleon of about 7.5 MeV. When it splits, the fission products are near the middle of the figure, where the binding energy per nucleon is in the neighborhood of 8.5 MeV. For each nucleon the binding energy goes up by about 1 MeV, for a total of more than 200 MeV each time a uranium nucleus undergoes fission. The nucleus doesn't just fall apart, it explodes, with the fragments sharing the released energy as kinetic energy.

A radioactive disintegration is also an explosion in which the products share the released energy as they separate, but the amount of energy that is released is usually of the order of at most a few MeV, much less than the energy that is released in fission.



Now look again at the graph of N against Z . The farther up a nuclide is on the graph, the greater is the excess of neutrons over protons. The fission process leads to a mixture of nuclides widely distributed near the middle of the graph, where the neutron excess is quite a bit smaller. If, for example, ${}^{238}_{92}\text{U}$ falls apart, with one of the fission products being ${}_{56}\text{Ba}$, the second must have an atomic number of $92 - 56$, or 36, the atomic number of krypton. The neutron excess of stable barium is at most 26, and for krypton 14, while for the parent uranium it is 54. There are lots of neutrons to spare! What happens to them?

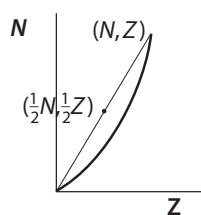
Some remain as part of the fission product nuclides. The number of extra neutrons is, however, so large that several fly off instantly as part of the fission process.

These neutrons may escape, or they may cause further nuclear reactions. If one of them hits another uranium nucleus, it may cause it to undergo fission. If enough neutrons are emitted each time, more than those that escape or cause reactions other than fission, there can be a *chain reaction*, with each fission process leading to further fissions. The chain reaction proceeds in a slow and controlled way in a *nuclear reactor*. It happens very quickly in an atomic bomb, which should, more correctly, be called a nuclear bomb.

EXAMPLE 9

Where are you most likely to find the fission products on the graph of N against Z ? What happens to them?

Ans.:



The heavier line on the figure represents the region of stability. Its curvature comes from the fact that the “neutron excess,” $N - Z$, increases as Z goes up. The straight thinner line shows that if a heavy nucleus with N neutrons and Z protons splits in half, to products with $\frac{1}{2}N$ neutrons and $\frac{1}{2}Z$ protons, they are substantially above the stability region, with too many neutrons.

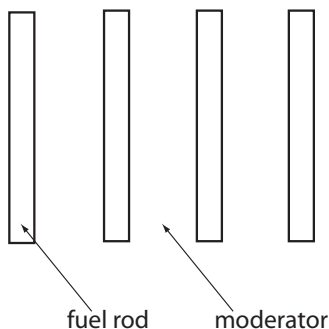
During fission a few neutrons are liberated and fly off separately. Negative beta decay reduces N by one and increases Z by one. It reduces the neutron excess and brings the nuclides closer to the region of stability. It is the most common form of radioactivity among fission products.

We know that neutrons can initiate nuclear reactions much more easily than protons or alpha particles because they are not deterred by the electric field, the *Coulomb barrier*, which repels positively charged particles. There is an additional, perhaps unexpected feature that comes into play: the slower a neutron moves, the more likely it is to cause nuclear reactions.

There are two reasons. One is that a slower neutron spends more time in the vicinity of the target nucleus and so has a greater chance to interact with it. The second one is that a slower neutron has a larger de Broglie wavelength. Where the neutron is cannot be localized more closely than by saying that it is within about a wavelength. In other words, the neutron will be able to initiate a reaction if it comes within a wavelength of the target nucleus.

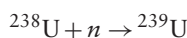
The neutrons that are emitted in the fission process have kinetic energies of the order of 1 or 2 MeV. To enhance the probability of further fissions, and that there will be a chain reaction, it is advantageous to slow down the emitted neutrons. This may be done by using a material that has little chance of undergoing a nuclear reaction itself, but that can slow down the neutrons by letting them bounce around among its nuclei.

A material used for this purpose is called a *moderator*. It will make the neutrons slow down most efficiently if its nuclei have masses close to that of the neutron. Hydrogen is the obvious first one to consider. To have a sufficient density it is used in the form of water. A drawback is that the proton can absorb a neutron to form a deuteron. Heavy water (in which the hydrogen is the isotope ${}^2\text{H}$) does not suffer from this limitation. It is present in all water, but deuterium is only 0.015% of natural hydrogen and is difficult and expensive to separate. The first nuclear reactor used carbon as a moderator in the form of graphite blocks alternating with the uranium fuel.



Natural uranium consists primarily of ^{238}U , with 0.72% of ^{235}U . Both can undergo fission when they absorb neutrons, but only the ^{235}U will do so with slow neutrons, i.e., neutrons that do not have an appreciable amount of kinetic energy. The slow neutrons have a much greater probability of causing fission. This makes ^{235}U the uranium isotope with the most favorable properties for a chain reaction. Its percentage in natural uranium is sufficient for a nuclear reactor that uses heavy water or carbon as the moderator. The majority of nuclear reactors today use water as a moderator, and in that case the uranium fuel needs to be *enriched* so as to contain a greater percentage (3–5%) of ^{235}U .

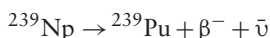
Later experiments showed that Fermi was right to look for elements beyond uranium when he bombarded uranium with neutrons. In addition to the fission products, which were responsible for most of the radioactivity, transuranic elements were also formed. Element 93 (neptunium, Np) and element 94 (plutonium, Pu) arise from the neutron absorption of ^{238}U :



The ^{239}U is formed in an excited state and emits a γ -ray as it goes to its ground state. It is radioactive with a half-life of 23 min and emits a β particle:



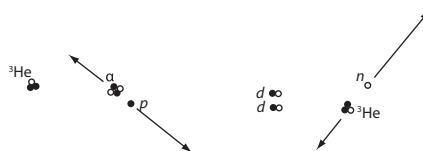
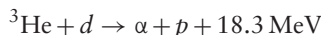
^{239}Np is also a β emitter. Its half-life is 2.4 days.



Plutonium 239 is an alpha emitter with a half-life of 24,000 years. Just like ^{235}U it undergoes fission when bombarded with slow neutrons.

Fusion reactions

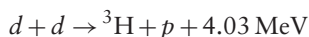
The binding-energy curve illustrates that the fission process leads to the release of energy: the binding energy per nucleon increases when the parent nucleus on the right side of the graph splits into two fission products near the middle. We see that the binding energy can also rise if we start from the left. This happens, for example, when helium 3 is bombarded by deuterons:



Such reactions are called *fusion* reactions. They would seem to be a great way to produce energy, but there are some major obstacles. The reaction of this example requires ^3He , which occurs in nature, but only to a minute extent (0.00014% of He). It can be made by nuclear reactions, but that is a slow and expensive process. There are other fusion reactions that use more common materials, such as



and



All fusion reactions share the difficulty that the nuclei repel, in other words that the Coulomb barrier has to be overcome. This can be done by using cyclotrons or other nuclear accelerators to give one of the initial materials sufficient energy. However, the machines require much more energy for their operation than is released in the reactions, so that their use does not lead to the production of net usable energy.

Another way to give the reacting nuclei more kinetic energy is to heat the material that contains them. With the growth of understanding of nuclear reactions in the 1930s, it became clear that this is how energy is produced in the sun and in the other stars.

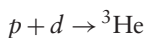
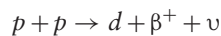
To produce the temperatures required for fusion reactions on earth is formidably difficult.

The kinetic energies of the atoms become so high that electrons leave them. In other words, the atoms become ionized. (An assembly of ionized atoms is called a *plasma*.) The major problem is that the ions are positively charged and repel each other. It is very difficult to keep them together long enough and at a high enough concentration for the fusion reactions to occur with a net release of energy.

The energy of stars

The main process by which energy is released in the stars is the *proton-proton cycle* (or p-p cycle) of nuclear reactions, whose net effect is to produce helium from hydrogen with the release of energy. The binding-energy curve shows that the energy that is liberated is about 6.2 MeV per nucleon, or about 25 MeV for each alpha particle that is produced.

The basic reactions of the cycle are



If we multiply the first and second reactions by two, and then add all the nuclei on the left and on the right, we see that the net effect is to change four protons into an alpha particle, two positrons, and two neutrinos.

EXAMPLE 10

Use the masses to determine the energy that is liberated in the first of the reactions of the p-p cycle. The mass of the deuteron is 2.013558 u.

Ans.:

The mass-energy balance for this reaction is $2m_p = m_d + m_e + Q$, where Q is the energy that is released. (The neutrino mass is either zero or negligibly small compared to the other masses.) $Q = 2(1.007276 \text{ u}) - 2.013553 - 0.000549 \text{ u} = 0.000450 \text{ u} = 0.42 \text{ MeV}$.

A feature of this cycle with vast consequences is that the first reaction proceeds at a much slower rate than the others. (This is related to the fact that this reaction involves electrons and neutrinos, which are not affected by the strong nuclear force. The reaction rate is

therefore determined by the much weaker *weak force*.) Hence it, more than any of the others, determines the rate at which the sun and the other stars radiate energy. The timing of the life cycles of stars and of the evolution of the universe is determined to a large extent by this reaction.

The p-p cycle takes the synthesis of nuclei from individual nucleons to $A = 4$, with helium 4. It seemed at one time as if all elements might have been created one step at a time from a primordial “soup” of nucleons through fusion reactions. To start with, the ratio of helium to hydrogen, as it is observed in the sun and the other stars, can be accounted for quite closely by that assumption. There is, however, a crushing obstacle to the creation of heavier nuclei: there is no combination of five nucleons that is stable. The formation of nuclei, one nucleon at a time, stops with helium 4!

Not only that, but there is also no stable nuclide with $A = 8$. ${}^8\text{Be}$ can be formed, but decays in about 10^{-16} s. When the gradual buildup of nucleons to form elements beyond helium was first considered, the absence of nuclides with $A = 5$ and $A = 8$ seemed like a minor problem that later knowledge would surely be able to deal with. That turned out not to be the case. The obstacle presented by these holes in the chart of the nuclides has remained.

How then did we get the rest of the periodic table of elements? The main reaction that passes the $A = 5$ hole turns out to be the two-step process by which ${}^8\text{Be}$ is first made from two alpha particles. It exists long enough to lead to ${}^{12}\text{C}$ when bombarded by another alpha particle. We can write the sequence of reactions as $3\alpha \rightarrow {}^{12}\text{C}$. It occurs only at extremely high temperatures, larger than about 10^8 K, and they are found only in the interior of stars.

The beginning of the universe and the cosmic background radiation

To the best of our knowledge, the universe, as we know it, started at a particular moment, about 14 billion years ago, with a “big bang.” The initial explosion created a mix of particles that soon (after a few minutes) became a mix of nucleons. Among the pieces of evidence that the universe originated in this way is the observed ratio of helium to hydrogen.

The most direct evidence is, however, the cosmic microwave background radiation, which was observed and identified in 1965. It had been predicted (by R. A. Alpher and R. Hermann in 1948) that the big bang should have resulted in radiation characteristic of the original temperature, which would then cool with the universe's expansion, so that the present spectrum would be like that of a radiating body at a few kelvins. The prediction was not, however, followed up for 16 years, partly because it was thought that the radiation would be too weak to be detected.

In 1964 R. H. Dicke suggested to his colleagues Peter Roll and David Wilkinson at Princeton University that they set up a microwave antenna to look for the radiation. They were getting their equipment ready when there was a phone call during a meeting of the group. When Dicke put the phone down he said: "Boys, we've been scooped!"

The call was from Arno Penzias at the Bell Laboratories in Holmdel, New Jersey, who, with Robert Wilson, had been testing a large and sensitive antenna. They observed radiation that they could not eliminate. It did not change with the direction in which the antenna was pointing, nor with the time of day.

Now anyone who has ever dealt with an electronic detection system, or at least turned on a radio, knows that there is always some unwanted background, referred to as "noise." The small amount of radiation seen by Penzias and Wilson would probably have been dismissed by most others. These two, however, thought that their antenna was so good that there should not be any spurious signals, no matter how small.

They puzzled over its possible origin, but had no clue until someone who knew of the Princeton group's activities suggested the fateful call. Two papers were published, one by Penzias and Wilson describing their observations and an adjoining one by the Princeton group on the likely source. Penzias and Wilson shared the Nobel Prize in physics for 1978.

A short time later the Princeton apparatus began to give results. It provided a point on the spectrum of the radiation, i.e., on the graph of intensity as a function of wavelength. Eventually the complete spectrum was measured and

shown to have the form expected for the radiation from a hot body at 2.7 K. Since then, largely through the efforts led by Wilkinson, the cosmic microwave background radiation has turned out to be a rich and unique source of information on the early universe. With increased precision it was seen not to be completely isotropic, but rather full of hot and cold spots that have been called "the scars of creation." They are the remnants of the original inhomogeneities at the beginning of the evolution that led to our present nonuniform universe.

13.8 Particles

By the time the neutron was discovered in 1932, all the basic constituents of matter seemed to be assembled. Nuclei consist of protons and neutrons and atoms consist of nuclei and electrons. This, however, turned out not to be quite the whole story. First the positron was discovered (also in 1932) as the antiparticle to the electron. It was realized that antiparticles might also exist for the proton and neutron, and when higher-energy accelerators were available they were indeed observed in 1955 and 1956.

A completely different particle was predicted in 1935 by Hideki Yukawa. It was based on the quantum-mechanical picture of the electromagnetic force as being "mediated" by photons. This means that the force can be described as an *exchange* of photons, a sort of throwing back and forth of photons between the two bodies that exert the force on each other. Yukawa tried to think of a particle that would play the same role for the strong nuclear force. He showed that the mass of such a particle should be inversely proportional to the range of the force. For the electromagnetic force (with its infinite range) the mass of the mediating particle (the photon) is zero. For the nuclear force he predicted a mass about 250 times as large as that of the electron.

There was great excitement in 1937 when a particle (now called the *muon*) with 207 times the electron mass was found, and it was disappointing when experiments showed that it has none of the other properties of Yukawa's particle. In particular, it is totally unaffected by the strong nuclear force. We know now that it is a particle

subject instead to the weak force. Its properties, except for its mass, are almost entirely the same as those of the electron. Its mean lifetime ($\frac{1}{\lambda}$) is 2.2×10^{-6} s and it decays into an electron, a neutrino, and an antineutrino.

The particle predicted by Yukawa was eventually observed in 1947. It is called the *pion*. It can be positive (π^+) or negative (π^-), with masses of 140 MeV and lifetimes of 2.6×10^{-8} s, decaying into a muon and a neutrino. There is also a neutral pion (π^0), with a mass of 135 MeV and a lifetime of 8×10^{-17} s, as it decays into two gamma rays.

Later (in 1983) particles mediating the weak force were identified, leaving only the particle mediating the gravitational force, the *graviton*, undetected.

Shortly after the pion was observed, other particles came on stage in bewildering profusion. They lived for small fractions of microseconds so that the thought that they were constituents of matter was hardly any longer appropriate. They did, however, provide a rich medium for experimental and theoretical exploration of regularities among them, to sort out what features are fundamental and which particles are truly “elementary.”

Eventually it became possible to classify them in families. Among the “old” particles we still think of the electron as elementary. That means that it has, as far as we know, no smaller constituents. In fact, it seems to have no measurable size at all. It is part of the family called *leptons* (“the light ones”), a name given to small Judaeo-Christian coins of biblical times. This name was more appropriate before the *tau* (τ) was discovered with a mass greater than that of the proton and neutron. The family consists of the electron, the muon, and the tau, each with its own kind of neutrino, six particles in all, plus their antiparticles. Their primary distinguishing feature is that they are subject to the weak but not the strong nuclear force. (There is also the gravitational, and, for the charged leptons, the electromagnetic force.) Insofar as the neutrinos have negligible mass, the weak force is the only one that they respond to. This explains why they hardly interact with anything and can go right through the earth without much chance of being stopped.

That the proton and neutron are not elementary particles is foreshadowed by their magnetic

properties. Both have the same spin angular momentum as the electron. They both also have magnetic moments. It is not surprising that the proton has a magnetic moment, but its magnitude is unexpected: it does not seem to have any recognizable relation to the angular momentum. The neutron also has a magnetic moment, and this is very strange, since it has no charge and a magnetic moment is always associated with a moving charge. The measured magnetic moments of both make sense only if we accept that both the neutron and the proton are composite particles. Their constituents must be charged, although in the case of the neutron the charges must add up to zero.

The two nucleons are part of a family called *baryons* (“the heavy ones”). A third family is composed of the *mesons*, of which the pion is one. Both the baryons and the mesons are primarily affected by the strong nuclear force; together they are called *hadrons*. None of them is an elementary particle. Today we believe that they consist of particles that are elementary, called *quarks*. A meson is made of a quark–antiquark pair. A baryon consists of three quarks. We know of six kinds of quarks, each with its antiquark. Each quark has a fractional charge that has a magnitude either $\frac{1}{3}$ or $\frac{2}{3}$ that of the electron. A difficulty is that it seems to be impossible to observe quarks individually, i.e., outside the particles of which they are parts. They have been “detected” only by high-energy scattering experiments.

There will, no doubt, be further developments and perhaps surprises. New insights have been gained each time higher energies have been achieved. More powerful devices continue to be planned, with the expectation that we may continue to learn more about the fundamental interactions that underlie the structure of the universe.

13.9 Summary

The interaction of an atom with its neighbors is through the electrons, but there can be no atom without a nucleus. Think of a baseball team: all action is by the players, but the manager controls where they are and what they do.

Actually the nucleus has much tighter control than any baseball manager. With one more

proton in it the electric field distribution is quite different, and so is the behavior of the electrons: we have a different element. But to change a nucleus is not so easy. It takes energies of the order of MeV, a hundred thousand to a million times as much as the energy to take away an electron.

To make it happen we need devices such as cyclotrons that can give charged particles energies that are sufficiently high to initiate *nuclear reactions*. Some nuclei disintegrate spontaneously. They are *radioactive*. By emitting alpha particles or beta particles they change into nuclei of other elements.

The nucleus of the atom determines which element an atom belongs to. Specifically it is the number of protons (Z), which is therefore called the atomic number. There can be different numbers of neutrons (N) in the nucleus (different isotopes), but different isotopes of the same element have the same chemical properties.

A given combination of Z and N represents a *nuclide*. Nuclides with low Z have values of Z and N that are close to each other. For higher values of Z there is a neutron excess because the electric repulsion between the protons favors neutrons over protons.

Some nuclides are unstable or *radioactive*. They fall apart (*decay*) spontaneously, with a characteristic half-life. They can emit alpha (α) particles (helium-4 nuclei consisting of 2 protons and 2 neutrons), beta rays or beta (β) particles (electrons), or gamma (γ) rays (photons emitted by a nucleus).

An energy-level diagram can represent the transition of a nucleus from a higher state to a lower state with the emission of a gamma ray. It can also represent a transition from one nuclide to another with the emission of an alpha particle. In that case Z decreases by 2 and N decreases by 2, while the alpha particle carries away 2 protons and 2 neutrons.

The emission of a beta ray carries away an electronic charge ($-e$) so that the remaining nucleus has one more positive charge ($+e$). It has one more proton and is therefore a nucleus whose Z has increased by one. The emission of the beta ray is accompanied by the emission of

a neutrino. The energy that is released is shared as kinetic energy by the beta ray and the neutrino. (The kind of neutrino that accompanies electron emission is called an *antineutrino*.)

Each radioactive nuclide has its characteristic half-life. A radioactive sample decays exponentially.

The total binding energy of a nucleus is the energy required to separate it into its protons and neutrons. It can be measured in mass units or in energy units, since the two are related by the expression $E = mc^2$. c^2 ($= 9 \times 10^{16}$ J/kg) can be considered a conversion constant: $1 \text{ kg} = 9 \times 10^{16} \text{ J}$. Another mass unit is the atomic mass unit (u), equal to $1.66 \times 10^{-27} \text{ kg}$; another energy unit is the MeV, equal to $1.6 \times 10^{-13} \text{ J}$. Both these and other mass or energy units can be used in mass and energy relations, but each term must be in the same units. For example, in the relation $m_x + E_B = Zm_p + Nm_n$, we can use MeV, kg, or u as long as the same unit is used for each of the four terms.

We can compare the stability of different nuclides on a graph of binding energy divided by the mass number (or nucleon number, $A = N + Z$) as a function of A . The higher a nuclide is on this graph, the more stable it is. The semiempirical binding energy relation fits this graph with terms whose form is in keeping with theoretical considerations and whose size is determined empirically. Its main terms are the volume energy, the surface energy, and the Coulomb energy. There is also the asymmetry term and the odd-even term. In addition there is a shell structure, which is not dominant as it is for electrons in atoms, but leads to particularly stable numbers ("magic numbers") for Z and N .

A charged particle emitted by a radioactive substance usually moves very fast. When it hits another material it gives its energy in small steps to the atoms of the material and causes the electrons in the atoms to go to higher energy levels (*excitation*) or to leave the atom (*ionization*). On the other hand, when a gamma ray hits a material it loses its energy in one big step by the photoelectric effect, the Compton effect, or if the energy is larger than $2m_e c^2$ (1.02 MeV), by pair production.

The *roentgen* (R) is a unit that measures the amount of ionization produced in air. The *gray* (Gy) and the *rad* are units that measure the energy absorbed per unit mass. The *sievert* (Sv) and the *rem* are similar, but are adjusted by an empirical *quality factor* (QF) to take into account the greater effect of some kinds of radiation, primarily alpha particles and neutrons.

An *absorbed dose* of 5 to 6 Sv is likely to be fatal. At the much lower levels of the natural background (3 mSv/year) it is very difficult to establish radiation effects. The question of how to extrapolate from higher doses is controversial.

Charged particles can be detected and counted by various kinds of counters that make use of the excitation and ionization that the particles produce. Neutral particles and gamma rays first give energy to charged particles that can then be detected and counted when they give rise to excitation and ionization.

Protons, alpha particles, and other nuclei can be used to initiate *nuclear reactions*. They have to have enough energy to overcome the electric repulsion of the target nucleus. Cyclotrons and other devices can give the particles the energy to come sufficiently close to get within the range of the nuclear force.

Neutrons do not face this *Coulomb barrier*. They can initiate nuclear reactions much more easily. But first they must themselves be the result of nuclear reactions to be liberated from nuclei.

Fission reactions occur in the heaviest nuclides. The result is that the nucleus splits into two pieces. Some neutrons are emitted at the same time. This is because the neutron excess ($N - Z$) is greater for heavier nuclides. These neutrons can initiate further fission reactions and so cause a *chain reaction*.

In a nuclear reactor the neutrons are slowed down by a *moderator* so that they have only the kinetic energy of their thermal motion. This makes them very efficient at causing fission in those isotopes that can undergo fission with slow neutrons, i.e., with neutrons that have little or no kinetic energy. Two isotopes that can undergo

fission when bombarded with slow neutrons are ^{235}U and ^{239}Pu .

Fusion reactions occur for the lightest nuclides. On the graph of E_B against A we see that the most stable nuclides are in the middle, near iron and nickel. Fission reactions proceed from the high- A end toward the middle, and fusion reactions from the low- A end. Energy is released in both cases. Neutrons can initiate fission reactions, but fusion reactions have to overcome the Coulomb barrier. One way to do this is to heat the reacting materials to very high temperatures. This is the source of the energy of stars.

The *microwave background radiation* is the best evidence that the universe as we know it started with an explosion (the “big bang”) about 14 billion years ago.

The basic building blocks of matter, the protons, neutrons, and electrons, were known by 1932 when the neutron was identified. Other particles have been discovered since that time, and the pursuit of their properties and functions is an important branch of today’s physics.

13.10 Review activities and problems

Guided review

1. What is the composition of a nucleus of ^{35}S ? Of ^{238}U ?
2. The mass of a $^{138}_{56}\text{Ba}$ nucleus is 137.874492 u. What is its total binding energy in MeV? What is the binding energy per nucleon?
3. The binding energy per nucleon for an alpha particle is 7.1 MeV. How much energy is made available in a process that synthesizes an alpha particle from its separate nucleons?
4. Describe how the graph of $\frac{E_B}{A}$ vs. A shows the stability of the stable nuclei. On this graph, at which value of A do you find the least stable nucleus and where is the most stable nucleus?
5. The mass difference between the nuclear masses of ^{235}U and ^{231}Th is 4.006524 u.

(a) What is the kinetic energy (in MeV) of the alpha particle emitted in the decay of ^{235}U ?

(b) What is the kinetic energy (in MeV) of the recoiling thorium nucleus? Can it be neglected?

6. The energy released in the β decay of ^{14}C is 0.16 MeV.

(a) What is the mass difference between the ^{14}C nucleus and its daughter nucleus (in MeV and in u)?

(b) What is the mass difference (in MeV) between the neutral atoms of ^{14}C and the neutral atom of the nuclide to which it decays?

7. What properties of the proton and the neutron cause one of these particles to be stable and the other to be radioactive?

8. A counter registers 10,000 counts per minute as it counts the radiation emitted by a sample of ^{131}I .

(a) What is the decay constant, λ , for ^{131}I ?

(b) What is the closest whole number of half-lives after which the counter (in the same position) will register less than one count per minute?

9. (a) Why is β^+ radioactivity rare among fission products?

(b) Where on the graph of N vs. Z are you most likely to find alpha radioactivity? What is the direction of a line on this graph from the parent to the daughter nucleus?

10. Use the nuclear masses to determine the energy that is liberated in the second and third steps of the p-p cycle. What is the total energy that is liberated in one cycle?

Problems and reasoning skill building

1. ^{61}Cu is radioactive and emits β^+ particles.

(a) Write the decay relation that shows the parent nucleus and the products of the disintegration.

(b) Write the relation between the masses of the parent and daughter nuclei and the other decay products.

2. A nuclide (in its ground state) is observed to emit alpha particles with energies of 3.7 MeV, 2.5 MeV, and 1.8 MeV.

(a) Draw a possible energy-level diagram for this decay.

(b) What other radiations do you expect to be emitted? Give their energies and show them on the diagram.

3. (a) After how many half-lives does a radioactive source decay to 10% of its original amount? Do this approximately by raising 2 to successively higher powers.

(b) Do it more precisely by using the relation $N = N_0 e^{-\lambda t}$ or its equivalent, $\lambda t = \ln \frac{N_0}{N}$.

4. ^{35}S is radioactive and emits β^- particles whose maximum energy is 0.17 MeV. What is the range of energies, from the smallest to the largest, of the electrons and of the neutrinos that are emitted in this process?

5. ^{90}Sr has a half-life of 30 years. 10 curies of this substance fall on a one-acre field. Draw a graph of the amount remaining against time for five half-lives.

6. Write the relations that describe the following radioactive transformations:

(a) the beta decay of ^{90}Sr ,

(b) the beta decay of tritium,

(c) the alpha decay of radon,

(d) the positron decay of ^{22}Na .

7. A radioactive substance has a half-life of one day.

How many days will it take for a counter that originally registers 100,000 counts per minute to have an average count rate of one count per minute?

8. The mass of an atom of $^{212}_{84}\text{Po}$ (i.e., that of its nucleus and all of the electrons) is 211.988842 u.

(a) What is the total binding energy of the nucleus?

(b) What is the binding energy per nucleon?

9. A deuteron and a tritium nucleus combine to form an alpha particle.

(a) What other particle is produced in this reaction?

(b) Use the masses to calculate the released energy.

10. ^{60}Co disintegrates by emitting negative beta rays with a maximum energy of 0.31 MeV.

(a) Write down the reaction relation.

(b) The beta ray emission leaves the daughter nucleus in an excited state from which it goes to the ground state with the successive emission of

gamma rays, first one of 1.17 MeV and then one of 1.33 MeV. Draw the energy-level diagram.

(c) What is the difference in the masses of the parent and daughter nuclei in units of u ?

11. The natural abundance of deuterium is 0.015%.

(a) What is the mass of heavy water in 1 m^3 of water?

(b) What is the number of deuterium atoms in this amount of water?

Multiple choice questions

1. The plot of N vs. Z for the stable nuclides deviates from the line $N = Z$ as a result of

- (a) the electric force,
- (b) the strong nuclear force,
- (c) the weak nuclear force,
- (d) the fission process.

2. The graph of $\frac{E_B}{A}$ vs. A for the stable nuclei has a peak at $A = 4$. This shows the following:

- (a) The alpha particle is more stable than any other nucleus.
- (b) The alpha particle is less stable than a nucleus with $A = 5$.
- (c) The alpha particle is more stable than a nucleus with $A = 3$.
- (d) The total binding energy of an alpha particle is greater than that of an iron nucleus.

3. Among radioactive radiations:

- (a) Alpha particles have discrete energies, but beta and gamma radiations have continuous ranges of energies.
- (b) Alpha and beta particles have continuous ranges of energies, but gamma rays have discrete energies.
- (c) Neutrinos and negative beta particles have continuous ranges of energies, but positive beta particles have discrete energies.
- (d) Alpha particles and gamma rays have discrete energies and beta particles have continuous ranges of energies.

4. There are 1000 protons and 1000 neutrons, each separate and with 0.1 MeV of kinetic energy, in a container. After one hour:

- (a) There will be more neutrons than protons.
- (b) There will be more protons than neutrons.

(c) There is no radiation that can escape the container.

(d) The neutrons will have changed into protons and the protons will have changed into neutrons.

Synthesis problems and projects

1. (a) What is the approximate whole number of grams in a mole of ^{226}Ra ? (You can also look up the more precise atomic mass.)

(b) What is the number of nuclei in 1 g of ^{226}Ra ?

(c) The half-life of ^{226}Ra is 1600 years. What is its decay constant, λ , in y^{-1} ? What is it in s^{-1} ?

(d) What is the activity (in Bq) of 1 g of ^{226}Ra ? (This is the rate of disintegration, $-\frac{dN}{dt}$, that is also equal to λN .)

(e) Compare your result to the number of Bq in one curie.

2. A hypothetical fusion plant uses the d - t reaction $d + t \rightarrow \alpha + n + 17.6 \text{ MeV}$. It produces 3000 MW ($3 \times 10^9 \text{ W}$) of thermal energy.

(a) What is the mass of heavy water that is required in one year to supply the deuterons?

(b) What is the mass of tritium that is required?

3. Tritium for the reactor of Problem 2 is made by the bombardment of ^6Li with neutrons.

(a) Write down the reaction relation.

(b) How many atoms and what mass of ^6Li are required for one year of operation?

4. What is the mass of ^{235}U that must undergo fission to produce the same amount of thermal energy as a ton (2000 lb) of coal and a barrel of oil? (The heat of combustion of coal is $1.3 \times 10^4 \text{ btu/lb}$ and of oil is $6.5 \times 10^6 \text{ btu/barrel}$.)

5. What mass of natural uranium is required in one year for a pressurized water reactor that produces 1000 MW of thermal energy from the fission of ^{235}U ?

6. The count rate from ^{14}C in a prehistoric wooden tool is found to be 6.5 counts per second. The mass of the tool is 220 g.

(a) Compare the ratio of ^{14}C to ^{12}C in this tool to the ratio found in living trees today, which is 1.3×10^{-12} .

(b) What is the age of the tool?

7. Radium 226 has a half-life of 1600 years. It emits an alpha particle with an energy of about 5 MeV.

(a) Write the relation that describes this reaction.

(b) What is the number of atoms in 1 mg (10^{-6} kg) of Ra?

(c) What is the total amount of energy (in J) that is emitted during one half-life by a source of radium whose initial mass is 1 mg?

8. The potassium content of the human body is about .12% and .012% of that is the radioactive isotope ^{40}K . The half-life of ^{40}K is 1.25×10^9 years. Each disintegration releases 1.3 MeV of energy, about 40% of which is absorbed by the body.

(a) What kinds of radiation are likely not to be absorbed by the body?

(b) What is the number of atoms of ^{40}K in a person whose mass is 70 kg?

(c) What is the decay constant of ^{40}K ?

(d) What is the number of nuclei of ^{40}K that disintegrate in the body of this person every second? What is the activity in microcuries?

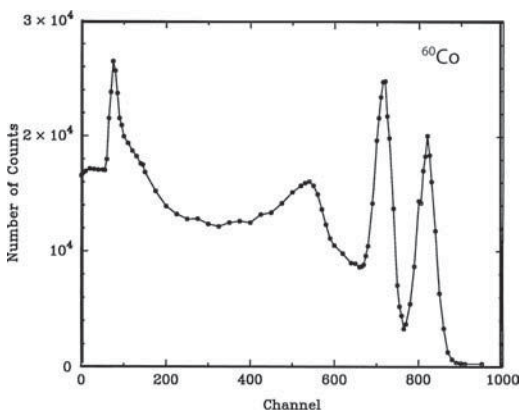
(e) What is the energy absorbed in the body per second and the energy per unit mass (in J/kg)? What is the absorbed dose in Gy and in rads?

What is the number of Sv and rems?

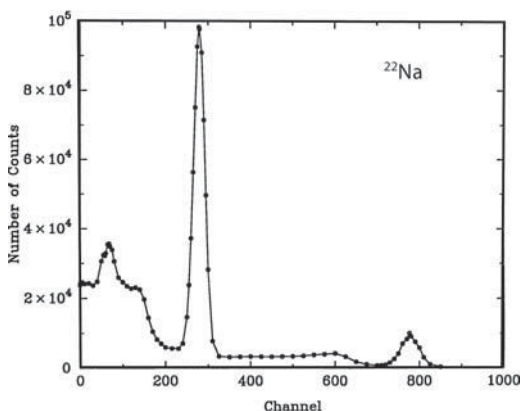
9. (a) Use the information given in Problem 6 in this section to estimate the mass and the activity (in Bq and Ci) of ^{14}C in a 70 kg person.

(b) The beta particles emitted by ^{14}C carry away 40% of the energy release in the disintegration. Calculate the absorbed energy and the energy absorbed per kg in the 70 kg person.

10.



(a) The figure shows the scintillation spectrum for ^{60}Co . The two peaks at the right are the “photopeaks,” resulting from the photoelectric effect produced by the two gamma rays at 1.17 MeV and 1.33 MeV in a sodium iodide crystal. Use the information on the graph together with that for the photopeak of ^{137}Cs in the text to draw a “calibration graph,” namely a graph in which you plot the channel number on the vertical axis and the energy on the horizontal axis.



(b) The spectrum in the text, for ^{137}Cs , and the ones here, for ^{60}Co and ^{22}Na were recorded with the same settings of amplification and voltages, so that they have the same relation between channel number and energy. You can now find the energy corresponding to the two photopeaks on the ^{22}Na spectrum. The one on the right is for a gamma ray. Find the energy for it from the channel number on your graph.

(c) Do the same for the large peak at the lower energy. The fact that ^{22}Na is a positron emitter tells you the origin of this peak. Its energy represents the rest energy of the electron.

(d) If you know the speed of light, you can consider your measurement to be that of the mass of the electron. Calculate the mass of the electron from your result.

(e) If you decide that you already know the mass of the electron from other experiments, you can consider your measurement to be one that determines the speed of light. What is the primary result of the experiment?

11. In Chapter 12 we said that the most probable angle between the incoming and outgoing photons in the Compton effect is 180° and that

the energy of the “backscattered” photon is then $\frac{EE_0}{2E+E_0}$, where E_0 is the rest energy of the electron. This angle also corresponds to the maximum energy that the Compton electron can have.

(a) Calculate the maximum Compton electron energy for ^{137}Cs and compare it to the scintillation spectrometer plot for this nuclide.

(b) Calculate the energy of the backscattered photon corresponding to the ^{137}Cs gamma ray and find it on the scintillation spectrometer plot. The peak corresponding to this energy is called the “backscattering peak.”

(c) What is the relation between the answers to parts (a) and (b)?