CHAPTER 4

Forces and Motion: Newton's Framework

What does it take to get something to move? You have to push a book to make it start to slide along the table. You have to throw a ball to make it leave your hand to fly through the air. The push on the book and that of your hand on the ball as you throw it are the forces that determine the motion. The book's motion depends not only on how hard you push, but also on the table and how smooth it is. The ball's motion also depends on forces other than that of your hand. Once the ball leaves your hand, the hand no longer exerts a force on it. The other forces continue to act: the earth pulls it down with the force of gravity. And on its way the air pushes against it and affects the path that it follows.

It's easy to think of more complicated examples. When you are on a bicycle, the downward push of your feet is linked to forces that make the bicycle move forward. And just think of all the forces that act in a moving car.

It took a long time for the relation between force and motion to be clarified. It was Isaac Newton, in the seventeenth century, who developed the framework that we still use today.

4.1 Newton's laws of motion

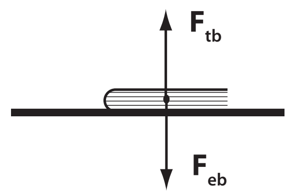
When the forces add up to zero: the first law

One of Newton's breakthrough contributions was to see that it takes no force at all just to keep an object moving in a straight line with constant speed. A nonzero *net* force is there only when the motion changes in speed or direction, in other words, when there is an acceleration.

Let's look at what happens when we slide a book along a table. At first it just sits there. We push it and it speeds up. We let go and it slides along by itself for a short distance. It slows down and comes to rest. On a smoother table it goes farther. On ice the same push makes it go quite far. In each case there is some friction, but the less friction there is, the farther the book moves. We can now imagine, as Newton did, that if there were no friction at all, the book would continue to move without losing speed. Today we can get quite close to that situation by letting an object move on a cushion of air, on an *air track* or *air table*. (You may also be familiar with a game called *air hockey*, in which a puck moves on a cushion of air, almost without friction.)

To make an object slide on a smoother and smoother surface is something we can do. It's an experiment. To make it move without any friction is something we cannot do. It's an *ideal* situation that we can only approach. Newton imagined what would happen in this ideal case, and concluded that if there were no friction, and no other horizontal force, the book would continue to move in a straight line with constant speed.

Are there any forces on the book when it just sits still on the table? Is the earth still pulling down on it? If the table were not there, the earth would pull the book down and it would fall to the floor. The table keeps it from falling, and while the earth pulls down, the table pushes the book up. The two forces, the force down by the earth and the force up by the table, are of equal size but in opposite directions. Their effects cancel each other out and the *net force* is equal to zero. Since there is no net force there is no change in the motion of the book.

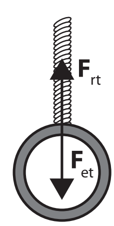


Each force is an *interaction*. It takes two! Whether it's the force of the earth on the book or the force of the hand on the ball, there are always two objects involved. The earth interacts with the book. The hand interacts with the ball.

When we write a symbol for *force*, we want it to tell us which two objects are interacting. We can write $F_{\text{earth on book}}$. To make that less clumsy we shorten it to F_{eb}. The second subscript stands for the object that we want to talk about, and the first for the other object that is exerting a force on it and so interacts with it.

EXAMPLE 1

A rope holds a tire as a swing on the playground. What are the forces on the tire?



Ans.:

The tire is pulled down by the earth with a force $F_{\text{earth on tire}}$ or F_{et}.

The tire is pulled up by the rope with a force $F_{\text{rope on tire}}$ or F_{rt}.

In the ideal case, when we imagine the book to slide without friction or other horizontal forces, the two vertical forces are still there and add up to zero. After your hand is no longer in contact with the book and it no longer exerts a force, there are no horizontal forces, since we assume that there is no friction. Since the two vertical forces add up to zero, and since there are no other forces, the sum of all the forces acting on the book, the *net force*, or the *unbalanced* force, is zero.

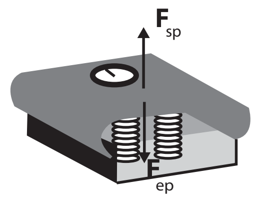
This is the situation described by Newton's first law of motion. To have no force on an object is an ideal situation impossible to achieve. But we can talk about what happens when the *net force* (the sum of the forces on the object) is zero: the object remains at rest, or if it is moving, it continues to move with constant velocity, i.e., in a straight line with constant speed. In either case there is no acceleration. *If the sum of all the forces on an object is zero, its acceleration is zero.* This is Newton's first law of motion.

What force really is: the second law

We have talked about forces from the beginning of this book. We already know a good deal about different kinds of forces. We know that there are four fundamental forces, namely gravitational, electric, and two kinds of nuclear. We know something about the electric forces between atoms, which lead to the forces exerted by springs and ropes, to friction and air resistance, and to the forces exerted by our hands as we push and pull.

But we haven't really said exactly what we mean by *force*. To say that it is a push or a pull was enough to get us started. Now we will use our preliminary and intuitive knowledge to develop a precise and quantitative definition. In the process we will also define what is meant by mass, and get to Newton's second law of motion.

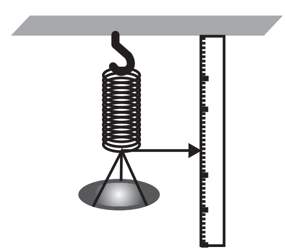
What happens when you step on a bathroom scale to weigh yourself? At least in an old-fashioned one there is a spring in it, which is compressed when you step on the scale.



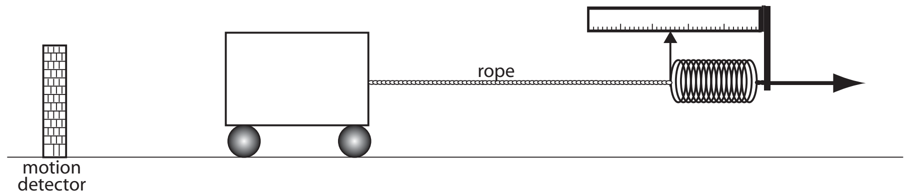
A pointer goes around a dial to tell you how much the spring is compressed. Two forces act on you as you stand on the scale: one is the force of the earth, pulling down on you ($F_{\text{earth on person}}$ or F_{ep}). This is the force that we call your *weight*. The other is the upward force of the scale with its spring ($F_{\text{scale on person}}$, or F_{sp}).

While you stand on the scale you have no acceleration. (Your velocity is constant and equal to zero.) That tells you that the net force on you is zero. The two forces on you must add up to zero.

The spring scale gives us a way to measure forces. We can also do that with a spring that is stretched. One end is attached to a fixed point, such as a hook on the ceiling or on a stand. From the other end we hang a pan on which we can place various objects to stretch the spring. A pointer is attached so that we can measure how far the spring has stretched.



Start with a set of identical metal blocks. Put one of them in the pan and mark a "1" on the scale next to the position of the pointer. With a second block the pointer moves further, and we mark a "2" where it stops, and so on. We then know how much any force stretches the spring.

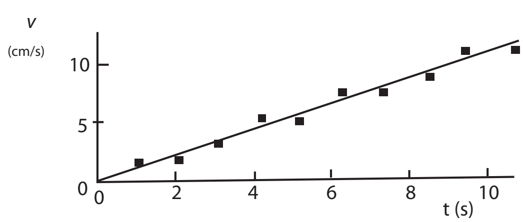


We say that the scale is now *calibrated* in units each of which is equal to the weight of one block.

This means that we now know what the pointer positions mean. When we take the blocks off and put on another object, such as a stone, the pointer moves to a new position. If it points to "4," we know that the weight of the stone is the same as the weight of four blocks. All we need to assume is that for a given weight on the scale, the pointer always returns to the same position. (This will be so as long as the spring is not stretched too far.)

Now let's do an experiment in which the object that is acted on by forces does not remain at rest. We can use a cart pulled with a rope, as on the diagram. If we attach our calibrated spring to the rope and pull on the spring, it will stretch and pull on the rope. The pointer position tells us the magnitude of the force with which the spring pulls on the rope and the magnitude of the force with which the rope pulls on the cart.

A sonic motion detector can measure the position of the cart at equal time intervals that are about 50 milliseconds apart. We can then use successive points to find the velocity, which can be plotted against time. Here is such a plot for a constant pulling force.



The graph is close to a straight line. Its slope is the acceleration, here equal to 1.11 in the units of the graph. If we repeat the measurements for different forces we find straight lines with different slopes, showing that the accelerations are different. We find that the relationship between the force and the acceleration is also represented by a straight line, showing us that the pulling force is proportional to the acceleration of the cart.

We can repeat the experiment with different numbers of blocks in the cart, but keeping the pulling force constant. As the number of blocks increases, the acceleration decreases.

To see what happens when we double the amount of material that is being pulled, we first determine the number of blocks that have the same weight as the cart. We can do this by using our spring scale. We find that doubling the amount of material being pulled by the same force leads to half the acceleration, tripling it to a third, and so on.

What property of the blocks determines how large the acceleration is? We call it their *mass*. More mass means less acceleration. We see that the acceleration is proportional to one over the mass (the reciprocal of the mass) that moves.

Now we're ready for a precise definition of force. We will take our preliminary and intuitive knowledge and the experimental results as guides. Only now we take the earlier statements to be exact: We saw that as the force was increased, the acceleration also increased. The graph showed that these two quantities are proportional. We also saw that as the mass was increased, the acceleration decreased. This time the graph showed that the acceleration is proportional to $\frac{1}{M}$, i.e., it is *inversely* proportional to the mass. We can combine these statements to say that the acceleration is proportional to $\frac{F}{M}$, i.e., that the net force is proportional to Ma.

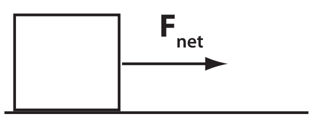
We still need to choose the units for measuring force and mass. For mass we use a standard mass, that of a particular metal cylinder kept in a laboratory in Paris, as the mass of one unit in the SI system. We call this mass one kilogram (1 kg).

In the SI system the quantity Ma is then measured in $\frac{\text{kg m}}{\text{s}^2}$. Since the units on both sides of an equation have to be the same, we let that also be the SI unit for force. We give it its own name, the *newton*, N. We can now define the net force to be equal to Ma. *The sum of all the forces acting* *on an object or system is equal to its mass times its acceleration.* This is Newton's second law of motion.

Guided by the experiments we have refined our previously rough idea of the meaning of the term *force*, and defined both mass and force. A net force brings about an acceleration. The larger the net force, the larger the acceleration. The two are *proportional*: ($a \propto F_{\text{net}}$). A net force of 100 N on an object produces twice the acceleration of a net force of 50 N.

The amount of the acceleration also depends on the mass of the object on which the force acts. More mass means less acceleration ($a \propto \frac{1}{M}$). The same force of 100 N produces an acceleration on a 5 kg object, which is twice as large as that which it produces on a 10 kg object.

EXAMPLE 2



A block of ice has a mass of 5 kg. The net force on it is 100 N to the right. What is its acceleration?

Ans.:
The relation between the three quantities is $F = Ma$, so that $a = \frac{F}{M} = \frac{100\,\text{N}}{5\,\text{kg}} = 20\,\text{N/kg} = 20\,\text{m/s}^2$.

Since both the force and the mass are in SI units (newtons, N, for force, and kilograms, kg, for mass), the acceleration comes out automatically in SI units, m/s^2.

Units

It's really important to keep track of units. It helps to use a *system* of consistent units. There are different metric systems and various English systems. In this book we will stay, for the most part, with the SI system, which uses kg, m, s, and N. Most countries have adopted these units and multiples of them.

We have used the *kilogram* as a unit of mass. Even in the United States it (and the *gram*, equal to 10^{-3} kg) is used on food labels. But it is unlikely that you have seen the *newton* mentioned outside of physics class. In common, nonphysics language it is pretty much unknown.

This seems surprising, since we talk about forces frequently. The most common force is the *weight*, the force with which an object is attracted to the earth. We can measure our weight by stepping on a bathroom scale, but you won't find any that are graduated in newtons, even in countries that use the SI system exclusively. Instead the weight will be marked and referred to in kilograms. How is that possible, when kilograms measure mass?

It's a sort of shorthand. We know that if an object has a mass of 1 kg, its weight, the force with which the earth attracts it, is 9.8 N. (This value is approximately the same at all points on the surface of the earth.) We can say that the weight is Mg, where $g = 9.8$ N/kg. This relation for the weight as equal to Mg has the same form as $F = Ma$. g is the acceleration of an object when the only force on it is its weight. The units of g are the usual units of acceleration, m/s^2, which are the same as N/kg.

If people say (incorrectly) "the weight of this book is 1 kg," what they mean is "the weight is that of a book whose mass is 1 kg." If you step on a metric scale and it reads 70 kg, it means that your weight is that of any object whose mass is 70 kg.

You might say that it tells you that your mass is 70 kg, but that is not necessarily so. If you take your bathroom scale (the kind that has a spring inside) to the top of a mountain, it will read less because the force with which the earth attracts you is then smaller. The weight is still Mg, but the value of g is now smaller because you are farther away from the center of the earth.

But your mass will not change! It is the quantity that tells you what your acceleration is when a force is applied to you, and that doesn't change, whether you are on earth, on the moon, or anywhere else. In other words, a spring scale graduated in kg will no longer read correctly if g is no longer 9.8 m/s^2, or 9.8 N/kg.

In the commonly used English system the units are defined differently. Here the unit of force is the *pound*, equal to 4.445 N. In this system it is the unit of mass that is almost never used.

The weight (on earth) of an object whose mass is 1 kg is 9.8 N, or 2.205 lb. We can say, more briefly, that a kg weighs 9.8 N or 2.205 lb. It is important to remember that the kg is a unit of mass, while the N and the lb are units of force.

EXAMPLE 3

Bob's mass is 70 kg. He is on the surface of the earth, where $g = 9.8\ \mathrm{m/s^2}$.

(a) What is his weight in newtons?

(b) What does a metric scale register when he steps on it?

(c) What is his weight in pounds?

Ans.:

(a) His weight is $Mg = (70\ \mathrm{kg})(9.8\ \mathrm{N/kg}) = 686\ \mathrm{N}$.

(b) The scale is graduated in kg, and registers 70 kg. His weight is that of an object whose mass is 70 kg. This weight is Mg or 686 N.

(c) Bob's weight in pounds is $(70)(2.205) = 154$ lbs.

Inertial mass, gravitational mass, and the principle of equivalence

Newton's law of gravitation says that there is a force of attraction between any two objects, i.e., there is a force on each, resulting from the gravitational attraction to the other. A curious feature now becomes apparent. We defined mass from $F = Ma$ as the quantity that tells us what the acceleration is when a given force is applied. Quite separately, Newton's law of gravitation says that mass is the quantity that tells us what the gravitational force on an object is. Two quite distinct physical phenomena are involved. Are we justified in using the same quantity (mass) and the same symbol (M) in both cases? It may be better to give them different names, *inertial mass* for the one based on $F = Ma$ and *gravitational mass* for the one based on gravitation. But no observation or experiment has ever been able to detect a difference.

In Newtonian, classical physics that's as far as we can go. The fact that the same quantity governs two seemingly unrelated phenomena remains a fortuitous quirk of nature.

Einstein, in 1915, saw more. Consider a closed box, or elevator, he said. Let go of an object in your hand, and it falls to the floor. That's not very mysterious. Presumably it is falling down because of the gravitational attraction of the earth. But you can't look out. Is there another explanation? Perhaps you are far from the earth and the elevator is accelerating. You feel the floor pushing up on you, and when you let go of the object in your hand, it accelerates with respect to the floor. There is no way, from inside the elevator, for you to tell the difference between the two explanations. The first case depends on gravitation and involves the gravitational mass. The second depends on acceleration and involves the inertial mass. But you can't tell the difference. They are the same.

This is the principle of equivalence, and it is the cornerstone and starting point of the *general theory of relativity*, the modern theory of gravitation.

To have unified the two aspects of mass would already be a great achievement. Beyond that, however, the general theory of relativity leads to results different from those described by Newton's law of gravitation. Einstein predicted three observational differences when he first described the theory in 1915. They are three astronomical effects. The first is a difference in the orbit of *Mercury* (the planet closest to the sun) from the Newtonian calculation. The second is the bending of light when it gets close to the sun. The third is a change in the wavelength of light emitted from a source where the gravitational force is very large.

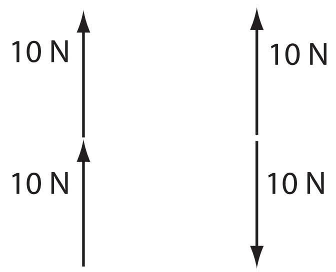
For each case the observations showed that Einstein's predictions were correct. In other words, the general theory of relativity was seen to describe the observed gravitational effects better than Newton's law of gravitation. For decades they remained the only observable results of the theory that were different, and came to be regarded as curiosities. With advances in observational astronomy in the last part of the twentieth century, however, a whole new era began for the general theory. After lying dormant for a long time, it is now a vital component of modern physical science.

One widespread application is in Global Positioning Systems (GPS). The great precision of today's global positioning devices relies on calculations using the general theory of relativity.

4.2 Adding forces: vectors

One dimension

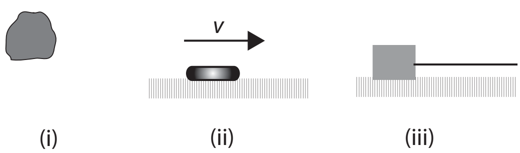
Let's talk again about the book at rest on the table. Suppose its weight is 10 N. That's the downward gravitational force attracting it to the earth. There is also the force with which the table pushes up. There is no acceleration, and therefore there is no net force. That means that the two forces (one up and one down) cancel. They are each 10 N, and they add up to zero. We see that we are dealing with quantities that behave very differently from numbers.



The direction matters. If two 10 N forces are in the same direction, their sum is 20 N. But if they are in opposite directions, they add up to zero. A quantity that has not only a size or magnitude, but also a direction, is a *vector* quantity. Examples are displacement, velocity, acceleration, and force. Quantities that have no direction, such as temperature, time, money, and energy, are scalar quantities. They add just like numbers.

When all the forces are vertical, we can say that we will let all upward forces be positive and all downward forces negative. (It could be the other way around. It's a choice we make. Either way, the physical result is the same.) The weight (the downward force that the earth exerts) is then −10 N, the upward force of the table is 10 N, and their sum is zero. The net force is zero even though there are two forces.

EXAMPLE 4



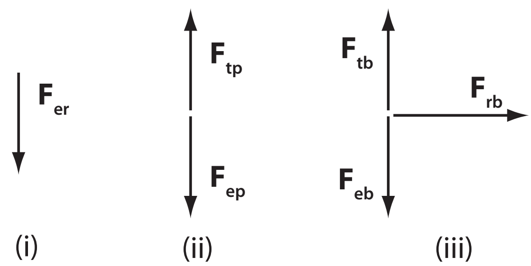
The figure shows (i) a freely falling rock, (ii) a hockey puck on ice, moving with constant velocity, and (iii) a box being pulled on a frictionless table.

For each of the objects in the figure,

(a) represent the forces on the object by vectors.

(b) Write the mathematical and the verbal statement of Newton's second law with these forces.

Ans.:



(i) $F_{er} = Ma$. The force of the earth on the rock (its weight) is equal to the rock's mass times its acceleration.

(ii) $F_{ep} - F_{tp} = 0$. The magnitude of the force of the earth on the puck (the weight of the puck) minus the magnitude of the force of the table on the puck is zero. There is no horizontal acceleration, and therefore no net horizontal force.

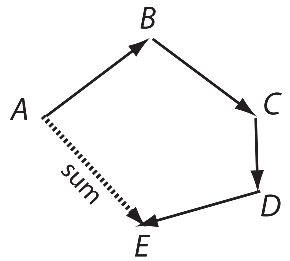
(iii) $F_{eb} - F_{tb} = 0$. $F_{rb} = Ma$.
Vertical forces: the magnitude of the weight of the box minus the magnitude of the force of the table on the box is zero.
Horizontal forces: There is only the force of the rope on the box. It is the net force and it is equal to the box's mass times its acceleration.

Two or more dimensions

Using only positive and negative numbers works as long as the forces are along the same line, up and down, or right and left. But what do we do if the forces are at some other angle to each other?

We start by representing each vector quantity by a line, whose length represents the magnitude, and whose orientation, with its arrow, shows the direction.

It's easiest to see the general method if we use displacement vectors. You just make a map. Suppose we look at the displacements from *A* to *B*, then from *B* to *C*, from *C* to *D*, and finally from *D* to *E*. The first vector goes from point *A* to point *B*, with its arrow pointing toward *B*. The other



vectors follow, "tail" to "head," until we get to point *E*. The sum of the four displacements is the single displacement from the beginning point (*A*) to the endpoint (*E*).

That's the way it works for all kinds of vectors. The most straightforward way to add them is to draw them so that they touch, head to tail. Their sum is the single vector from the beginning (tail) of the first to the end (head) of the last. (If you make a mistake and let two heads touch, you'll get the wrong answer!)

Go to the PhET website (http://phet.colorado.edu) and open the simulation *Vector Addition*.

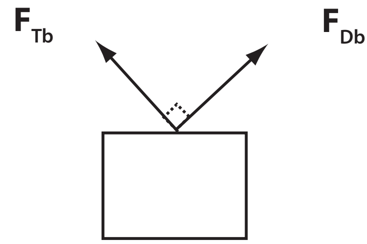
Check "Show Sum" and "None" under Component Display. The arrows representing vectors can be dragged from the basket. Their length can be changed and they can be tilted by grabbing their heads. Explore different angles and different numbers of vectors. Look at the sum when you (correctly) put two vectors head to tail as well as when you put them tail to tail and head to head. Tilt a vector and observe what happens to the vector that represents the sum.

Come back to this simulation later after the discussion of vector components.

A vector is usually described by a boldface symbol (**A**) or a symbol with an arrow over it ($\vec{A}$). The plain symbol then refers to the magnitude (or length) only. If the vector nature of a quantity is obvious we may leave out the vector notation. But we have to be careful. For example, $A+B$ represents the sum of the magnitudes or lengths of the two vectors **A** and **B**. But $\mathbf{A}+\mathbf{B}$ is a vector whose magnitude is less than $A+B$ unless the two vectors point in the same direction. For example, if **A** is 10 N to the right, and **B** is 10 N to the left, the vector sum $\mathbf{A}+\mathbf{B}$ is equal to zero, but $(A+B)$ is the sum of the magnitudes of the two vectors, and is equal to 20 N.

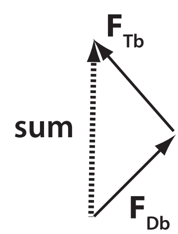
If the three vectors $\mathbf{F_1}, \mathbf{F_2}, \mathbf{F_3}$ represent all of the forces that act on an object, then their vector sum, $\mathbf{\Sigma F} = \mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3}$, is the net force, and this is the quantity that is equal to $M\mathbf{a}$. (The symbol Σ is a capital Greek *sigma*. It is often used to represent the word *sum*.) $\mathbf{\Sigma F}$ is the vector sum of all the forces acting on an object.

EXAMPLE 5



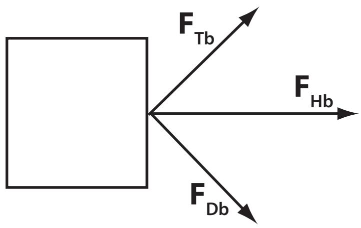
Tom and Dick pull on strings attached to a box with forces of 30 N each. (Call them F_{Tb} and F_{Db}.) They pull at right angles to each other, as shown in the diagram, which shows the view from above. What is the sum of the two forces?

Ans.:
Draw vectors representing the two forces head to tail:



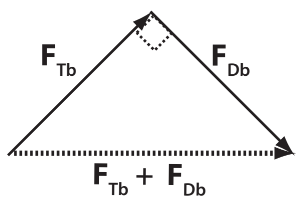
Their sum is the vector from the tail of the first to the head of the second. The two vectors and their sum form a right-angle triangle. The sum of the squares of the sides next to the right angle is equal to the square of the third side (the hypotenuse), i.e., the sum is $\sqrt{30^2+30^2}$ or 42.4 N. The sum of the two forces is therefore a force whose magnitude is 42.2 N, acting at an angle of 45° to the direction in which Tom pulls, as shown in the diagram. Note that the answer is the same if you start with Dick's force.

EXAMPLE 6



Tom, Dick, and Harry pull on strings attached to a box. Harry pulls to the right with $F_{Hb} = 40$ N. Tom and Dick pull with forces of 30 N each at angles of 45° to the direction in which Harry pulls, as shown in the diagram. What is the sum of the three forces?

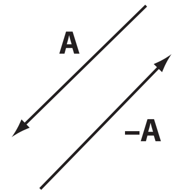
Ans.:
Start by adding Tom and Dick's forces, which we will call F_{Tb} and F_{Db}. Since they are at 45° to Harry's force, they are at 90° to each other. To add them we redraw the vectors head to tail.



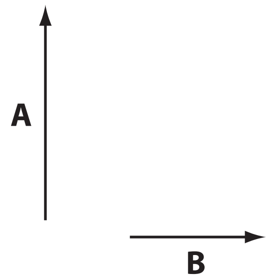
Their sum is the third side of the right-angled triangle formed by the two forces. It is to the right, with a magnitude of $\sqrt{30^2 + 30^2} = 42.4$ N.

Harry's force is in the same direction, to the right. We are left with two forces, 42.4 N and 40 N, both to the right. Since they are in the same direction, we can just add their magnitudes, to get 82.4 N to the right.

Since two vectors of equal magnitude and opposite directions add up to zero, we see that to change the direction of a vector is to make it the negative of the original vector. If we want to subtract **B** from **A**, we can rewrite **A** − **B** as **A** + (−**B**), and add the vector **A** to the vector −**B**, which is in the opposite direction to the vector **B**.



EXAMPLE 7



The vector **A** is 50 N north.

The vector **B** is 30 N to the east.

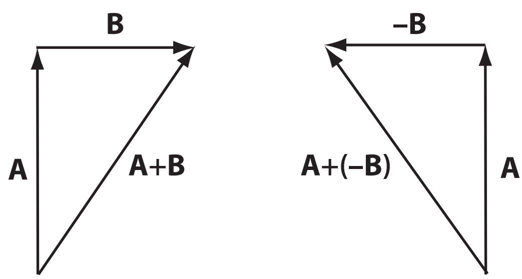
Draw the appropriate vector diagrams and

(a) find **A** + **B**

(b) find **A** − **B**.

Ans.:

(a) The magnitude of the sum is $\sqrt{50^2 + 30^2} = \sqrt{2500 + 900} = \sqrt{3400} = 58.3$ N in the direction shown in the diagram. (We can use the Pythagorean theorem, $A^2 + B^2 = C^2$, because the vectors are at right angles.)



(b) The vector −**B** is 30 N west. Add **A** and −**B**.

The magnitude is the same as for part (a). The direction is now different, as shown in the diagram.

We have used the symbol F for force in Newton's second law, as if there were only one force acting on our object. In general, we should use the *net* force, i.e., the vector sum $\Sigma\mathbf{F}$ of all the forces acting on the object. Newton's second law can then be written as $\Sigma\mathbf{F} = Ma$: *When one or more forces are applied to an object whose mass is M, and the vector sum of all the forces acting on the object is* $\Sigma\mathbf{F}$, *the acceleration of the object is given by the relation* $\mathbf{a} = \frac{\Sigma\mathbf{F}}{M}$.

Force diagrams

When we want to apply Newton's second law to a particular situation, we first have to decide which object we want to apply it to. Then we have to look for all the forces acting on the object. That means that we are not looking for the acceleration, or for the velocity, or for forces acting on other objects! We make a diagram on which we represent the object by a dot, and draw vectors representing all of the forces on the object. The important thing is to make sure that we put on the diagram only the forces acting on that particular object, and that we put on all of them. The result is a *force diagram*.

If you are not sure whether a certain force should be included, ask yourself: what is the object that I am considering? What are the other objects that are interacting with it? Are they touching? The other object that exerts a force can be some distance away if the force is gravitational or electric, but it has to be touching for a force like the tension of a rope or the push of a hand or of a spring.

We can now add up the vectors representing the forces so as to find their sum, $\Sigma\mathbf{F}$, and use it in the expression for Newton's second law. If there are several forces in the x direction, and several forces in the y direction perpendicular to it, it is often helpful and convenient to apply Newton's second law separately in the two directions: $\Sigma F_x = Ma_x$ and $\Sigma F_y = Ma_y$.

EXAMPLE 8

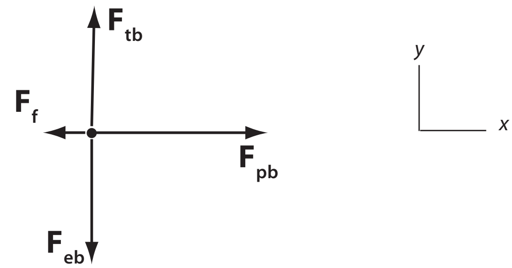
A book whose mass is 1.2 kg is on a table. It is being pulled forward by a horizontal force of 7 N. There is also a force of friction of 2 N.

(a) Draw the force diagram, labeling all the forces.

(b) Find the book's acceleration.

Ans.:

(a) On the force diagram the dot represents the book. Four forces act on it: Its weight, $F_{eb} = Mg$ (the force of the earth on the book), F_{tb}, the upward force exerted by the table on the book, F_{pb}, (person on book) the horizontal pull on the book to the right of 7 N, and F_f, the force of friction.



(b) Let the up direction be the y direction and the direction in which the 7-N force acts, the x direction.

The two forces along the y direction are F_{eb} and F_{tb}. There is no acceleration in the y direction. (The book does not jump up from the table.) Therefore the sum of the two forces along the y direction must be zero.

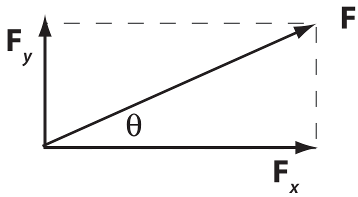
There are two forces along the x direction, the force of 7 N to the right and the force of friction (2 N) to the left, so that their sum is 5 N to the right. This is also the sum $\Sigma\mathbf{F}$ of all four forces, and it is therefore equal to $M\mathbf{a}$. $\mathbf{a} = \frac{\Sigma\mathbf{F}}{M} = \frac{5\,\text{N}}{1.2\,\text{kg}} = 4.17\,\text{m/s}^2$. to the right. (Since the force and the mass are both given in SI units, the acceleration is automatically also in the same system of units.)

We have added four forces. Two are along the y-axis and add up to zero. There is a net force along the x-axis. It is also the sum of all the forces acting on the book, and is therefore equal to Ma.

Vector components

Together the x-axis and the y-axis are a *coordinate system*. It allows us to describe the positions of points, lines, or objects. It is an aid to describing a situation or solving a problem, and we can choose it any way we want. We choose one direction of each axis to be positive and the other to be negative, Many times we choose the axes to be horizontal and vertical, but that is not always so.

If a force is at an angle to the axes, we can replace it with two forces that are parallel to the two axes and that add up to it. The one parallel to the x-axis is called the x component, F_x, and the one parallel to the y-axis is called the y component, F_y. For an angle θ between the direction of **F** and the x-axis, $F_x = F\cos\theta$ and $F_y = F\sin\theta$.



Adding vectors head to tail, as we have done so far, is convenient graphically. It also allows the calculation of numerical values of the magnitude and direction of the sum in simple cases, as, for example, when the vector diagram is a triangle with a right angle. In more complicated cases it may be better to use components.

To do this it is first necessary to choose a coordinate system, i.e., directions for the x- and y-axes. We can then find the x and y components of the vectors that are to be added.

All the x components can now be added. Since they are all along the same line, no angles need to be considered. It is important, however, to make sure which of the components are in the same direction as the x-axis, and are positive, and which are in the opposite direction and are negative. The x components are negative when the angle of the vector with the x-axis is between 90° and 270°, i.e., when the vector is in the second or the third quadrant. We can write the sum of the x components as ΣF_x. This quantity is the x component of the sum of the vectors.

Similarly the y components can be added to give ΣF_y, which is equal to the y component of the vector sum. A y component is negative when the angle of the vector with the x-axis is between 180° and 360°, i.e., when the vector is in the third or the fourth quadrant.

Knowing the x and y components of the vector sum gives us all of the information about the sum. We can calculate its magnitude, $\sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$. The angle that the sum makes with the x-axis is the angle whose tangent is $\frac{\Sigma F_y}{\Sigma F_x}$.

More on friction

Let's look again at the forces on the book on the table. When the book is at rest, without any horizontal forces, there are only two: one is the weight, Mg, for which we have also used the symbol F_{eb}, to indicate that it is the force of the earth on the book. It is always there, unless the object is far from the earth. Near the surface of the earth the value of g is about 9.8 m/s^2, or 9.8 N/kg. At some distance from the earth the weight is still Mg, but the value of g is then different. The direction of this force is toward the center of the earth.

The second force is the force of the table on the book, F_{tb}. It is upward and perpendicular to the surface of the table. This force is often called the *normal* force, F_n. (This does not mean that other forces are *abnormal*. The word *normal* here means the same thing as *perpendicular* or *at right angles*.) For a book sliding along an inclined plane, or for a car along a road or track at some angle, there is always a *normal* force, a force of the surface on the book or the car, at right angles to the surface along which it moves.

Now push the book horizontally with a force F_{pb} (person on book). At first the book does not move. There is a force of friction (F_f) in the opposite direction to F_{pb}, and it has the same magnitude as F_{pb}.

When we increase our push, F_{pb}, the force of friction initially grows with it, and the book remains at rest. There is, however, a maximum value beyond which F_f cannot grow, so that there is then a net force $F_{pb} - F_f^{max}$, and the book accelerates.

Once the book starts to move, the force of friction remains steady, but at its sliding value F_f^s, which is smaller than F_f^{max}.

The ratio $\frac{F_f^{max}}{F_n}$ is called the *coefficient of static friction*. The ratio $\frac{F_f^s}{F_n}$ is called the *coefficient of sliding friction*. Both of these coefficients depend on the nature of the two surfaces that are in contact. The symbol μ (Greek *mu*) is generally used for the coefficient of friction.

EXAMPLE 9

Go to the PhET website (http://phet.colorado.edu) and open the simulation *Forces and Motion*. Play with the *Introduction* and *Friction*. Look at the force diagram (here called "Free Body Diagram.") Then go to *Force Graphs*. Choose the small crate, friction off ("Ice"), check applied force, and click on all graphs (a, v, and x). Type in an applied force of 200 N, put the crate at −7 m, and record.

(a) What is the acceleration shown on the graph? Compare it to the value you expect from Newton's second law.

(b) What is the shape of the velocity graph? What is its equation? Calculate the velocity just before it hits the wall. Compare this value to the value on the velocity graph.

(c) What is the shape of the position graph? What is its equation? Calculate the position at the end. Compare it to the value on the position graph. (Don't forget where you started.)

(d) Clear. Check friction "wood." Check only the applied force graph. Press the "go" button.

Increase the applied force until the file cabinet moves. Find the necessary force to the nearest 10 N.

What is the coefficient of static friction?

(e) Once the object moves the force of friction decreases and the coefficient of friction decreases to the coefficient of sliding friction.

What happens to the force of friction when the object moves?

Find the coefficient of sliding friction.

Ans.:

(a) $a = 0.5\ \frac{m}{s^2}$.

(b) $v = at,\ v = 0.5t$.

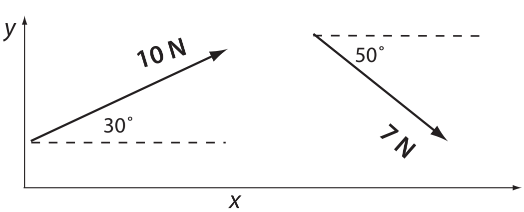
(c) Since he starts at $x = 7$ m, $x = -7 + v_0 t + \frac{1}{2}at^2$, i.e., $x = -7 + 0.25t^2$.

(d) The coefficient of static friction is $\frac{490}{100\times 9.8} = 0.5$.

(e) The coefficient of sliding friction is $\frac{294}{100\times 9.8} = 0.3$.

EXAMPLE 10

Find the x and y components of these vectors, using sines and cosines.



Ans.:

For the vector on the left, $F_x = 10 \cos 30° = (10)(0.866) = 8.66$ N.

$F_y = 10 \sin 30° = (10)(0.500) = 5.00$ N.

For the vector on the right, $F_x = 7 \cos 50° = (7)(.643) = 4.50$ N.

$F_y = 7 \sin 50° = (7)(.766) = 5.36$ N.

This is the *magnitude* of F_y. It points downward, in the $-y$ direction, and is therefore negative, equal to −5.36 N. (Similarly, an x component in the negative x direction would be negative.)

EXAMPLE 11

Go to the PhET website (http://phet.colorado.edu) and open the simulation *Ramp: Forces and Motion*. Play with the *Introduction*, Then go to *Force Graphs*,

(a) Check "wood." Choose the crate and use the position slider to put it on the ramp. Press the "go" button.

Change the angle of the ramp (use the angle slider or drag the top of the ramp) until you find the largest angle at which the crate just rests without moving down.

At this angle, what are the forces on the crate?

What is their sum?

One force is the weight. How are the other forces related to the weight?

What is the magnitude of each of the forces?

(b) Set the applied force to 300 N. Do not change the angle. Answer all the other questions in part (a).

(c) Set the applied force to 1000 N. Answer all the questions in part (b).

Ans.:

(a) The angle is 26.6°. The vector sum of the forces is zero. The weight is $Mg = 980$ N. The normal force is $N = Mg \cos\theta = 876$ N. The force of friction is $\mu N = 438$ N.

(b) With an applied force of 300 N the crate still does not move. The sum of the forces is zero. The perpendicular components are N and $Mg \cos\theta$. The magnitude of the applied force is equal to the force of friction plus the component of the weight $Mg \sin\theta$. Note that the force of friction is not equal to μ N. μ N is the maximum force of friction.

(c) With the applied force of 1000 N the force of friction is not large enough to keep the crate from moving. The sum of the forces is now not zero. The perpendicular components are as before and add up to zero. In the direction parallel to the ramp, using "up the ramp" as the positive direction, the net force F is equal to the applied force, F_a, minus the force of friction, f, minus the component of the weight down the ramp: $F = F_a - f - Mg \sin\theta$. The crate accelerates up the ramp with $a = \frac{F}{M}$.

EXAMPLE 12

Repeat Example 8 for the same book, but this time the book is being pulled forward by a string at an angle of 15° with a force of 7 N.

(a) Draw the force diagram.

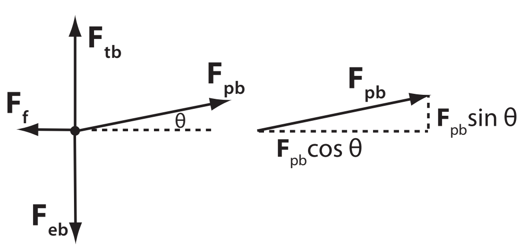
(b) What is the coefficient of friction in Example 8?

(c) Assume that the coefficient of friction remains the same, and find the force of friction.

(d) Find the book's acceleration.

Ans.:

(a) We choose the same coordinate system, the x-axis horizontal and the y-axis vertical. The force pulling the book ($F_{pb} = 7$ N) is now at an angle, and we can decompose it into its two components. Its x component is 7 cos 15°, or (7)(.97) = 6.76 N. The y component is 7 sin 15° = (7)(.26) = 1.82 N.



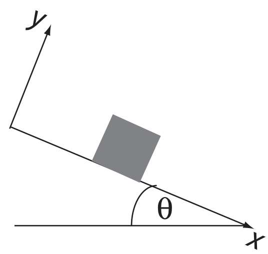
(b) The normal force, F_n, which is the force of the table on the book, F_{tb}, is different from its value in Example 8. It is $F_{eb} - F_{pb} \sin\theta$, i.e. (1.2)(9.8) −1.82, which is 11.76 − 1.82 or 9.94 N.

(c) In Example 8 the coefficient of friction is $\frac{F_f}{F_n} = \frac{2}{11.76} = 0.17$. In this example $\frac{F_f}{F_n}$ is again 0.17, but $F_n = 9.94$ N. Hence $F_f = (9.94)(0.17) = 1.69$ N.

The vertical forces add up to zero, as before, because there is no vertical acceleration. $\Sigma F_x = F_{pb} \cos\theta - F_f$, or 6.76−1.69, which is 5.07 N.

(d) The acceleration is $\frac{5.07}{1.2} = 4.23$ m/s^2.

EXAMPLE 13



A box is at rest on a surface that slopes at an angle of 30° to the horizontal.

(a) List the forces on the box. Draw a force diagram.

(b) What forces are perpendicular to the surface? What forces are parallel to the surface?

(c) The weight is neither parallel nor perpendicular to the surface. Use a coordinate system with the x-axis parallel to the surface and down it, and the y-axis perpendicular to the surface and up. Find the weight's x component and its y component.

(d) Write down the relation between the y components of all the forces.

(e) Write down the relation between the x components of all the forces.

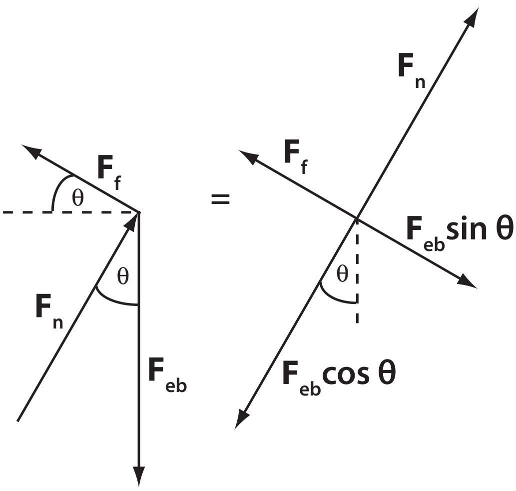
(f) The mass of the box is 2.5 kg. Find the magnitude of each of the forces.

Ans.:

(a,b) The weight, $F_{eb} = Mg$, is straight down, and therefore neither parallel nor perpendicular to the surface. The normal force, F_n, is perpendicular to the surface, and the force of friction, F_f, is parallel to the surface. (You can see that the two angles marked θ on the left are the same by imagining the two vectors F_{eb} and F_n rotating together through 90°.)

(c) The y component of the weight is $Mg \cos 30°$. The x component is $Mg \sin 30°$.

(d) $Mg \cos 30° - F_n = 0$.



(e) $Mg \sin 30° - F_f = 0$

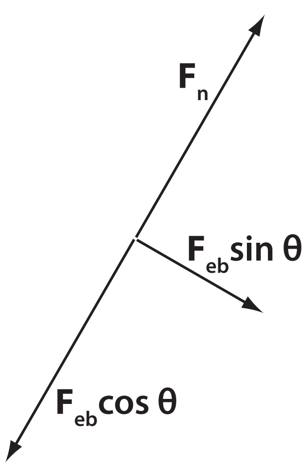
(f) $Mg = (2.5)(9.8) = 24.5\,\text{N}$.
$F_n = Mg \cos 30° = 21.2\,\text{N}$.
$F_f = Mg\, sin\, 30° = 12.25\,\text{N}$.

EXAMPLE 14

(a) The box of the previous example slides down the same surface without friction. What is its acceleration?

(b) The box slides down the surface with an acceleration of $2\,\text{m/s}^2$. What are the magnitude and direction of the force of friction?

Ans.:



(a) This time F_f is zero. The other two forces are unchanged. There is again no acceleration along the y direction. The force along the x direction is the x component of the weight, 12.25 N. There is no other force with a component in this direction, so that $a = \frac{12.25}{2.5} = 4.9\,\frac{\text{m}}{\text{s}^2}$.

(b) The force of friction is in the direction opposite to the direction of motion, in the $-x$-direction. The forces perpendicular to the surface are unchanged. The forces parallel to the plane, in the x-direction, are $Mg \sin 30°$ in the positive direction and F_f in the negative direction. $Mg \sin 30° - F_f = Ma$. $F_f = Mg \sin 30° - Ma = 12.25 - 5 = 7.25\,\text{N}$.

EXAMPLE 15

You jump from a table straight down to the floor. Neglect air resistance. Assume that your mass is 65 kg.

(a) Draw a force diagram for the time that you are in the air.

(b) What objects interact with you? What is the net force on you?

(c) What is your acceleration?

Ans.:

(a)

(b) Since we are neglecting air resistance, the only force on you is your weight, F_{ep}. It is equal to Mg or $(65\,\text{kg})(9.8\,\text{m/s}^2) = 637\,\text{N}$.

(c) Since the only force on you is your weight, your acceleration is $g = 9.8\,\text{m/s}^2$.

EXAMPLE 16

You jump from a table, as in the previous example, but this time you push off so that you jump away from the table and not straight down. Repeat parts a, b, and c of the previous example.

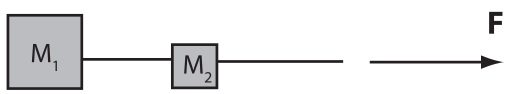
Ans.:

This time your path is not along a straight vertical line. However, as long as we continue to neglect air resistance, the only force that acts on you is still your weight. The answers to all three parts of this question are therefore the same as before. Your weight is the same, and your acceleration is g.

Object or system?

We often apply Newton's second law to a combination or *system* of more than one object. In fact, unless we are talking about a single particle without internal structure, every object consists of more than one particle. In the previous section we said that we first have to decide to which object we want to apply the law. Now we see that it is really the *system* that we have to choose. Only then can we decide what the forces are that we need to consider, namely all the forces that act on the system that we have chosen. To make clear what the system that we choose is, we can draw a dotted line around it on our sketch.

EXAMPLE 17



Two blocks are connected by a string. Their masses are 5 kg and 3 kg. The less massive one is being pulled by a second string, with a force, F, so that its acceleration is $2\,\text{m/s}^2$. (Neglect friction and all other forces. The vertical forces add up to zero and need not be considered.)

(a) Draw force diagrams for the system consisting of both blocks, and for each block separately (for the horizontal forces only).

(b) What is the force (the "tension") of the second string?

(c) What is the tension in the string connecting the two blocks?

Ans.:

(a)

Let F_{s1} be the force of the string on M_1 and F_{s2} the force of the same string on M_2. These two forces have the same magnitude but are in opposite directions. (This is an approximation. We are neglecting the mass of the string.) The force diagrams are for the system consisting of both blocks, the system with only M_1, and the system with only M_2.

(b) Apply Newton's second law to the system consisting of both blocks. There is only one force on this system, the force F. The mass of the system is the sum of the two masses, $5 + 3 = 8$ kg. Since the acceleration is $2\,\text{m/s}^2$, $F = Ma = (8)(2) = 16\,\text{N}$.

(c) The only force acting on the 5 kg mass is the tension in the string that connects the two masses. It is the force exerted by the string on the 5 kg mass, and we have called it F_{s1}. $F_{s1} = Ma = (5)(2) = 10\,\text{N}$.

The string connecting the two blocks pulls to the right on the 5 kg block with the force F_{s1} and to the left on the 3 kg block with a force F_{s2} of the same magnitude.

We know from part (b) that the force to the right on the 3 kg block is 16 N. We can use the forces on this block to find F_{s2} (and F_{s1}) in a second way. The net force is $16 - F_{s2}$ to the right. It is equal to Ma.

$16 - F_{s2} = Ma = (3)(a) = (3)(2) = 6$. $F_{s2} = 16 - 3a = 16 - 6 = 10\,\text{N}$, as before.

4.3 Momentum and its conservation. Action, reaction, and Newton's third law

When two objects collide, there are two quantities that we need to focus on. One is how fast they are going, the other is their mass. We define a new quantity, the *momentum*, equal to the product of the mass and the velocity. Like velocity, it is a vector quantity. A small car, moving slowly, has a much smaller momentum than a truck barreling along at great speed. A table-tennis ball coming at you fast is not likely to hurt you. A car with the same speed is vastly more dangerous.

Acceleration is the quantity that tells us how fast the velocity is changing. It is the rate of change with time of the velocity. Force (mass times acceleration) is therefore the quantity that tells us how fast the momentum (mass times velocity) is changing. It is the rate of change with time of the momentum. In symbols we can write $\mathbf{a} = \frac{d}{dt}(\mathbf{v})$, $\mathbf{F} = M\mathbf{a} = \frac{d}{dt}(M\mathbf{v})$. We can rephrase Newton's second law to say that the force on an object is equal to the rate at which the object's momentum changes.

If there is no net force on an object then its momentum does not change, it is *conserved*. At first sight this hardly seems like a new or important statement. Isn't that just Newton's first law, $a = 0$ when $F = 0$? Yes, that's true if we talk about a single particle, but look at what happens if we talk about two particles. Together we can think of them as a composite object, or a *system*. If there is no force on this system from outside it, its momentum is constant.

EXAMPLE 18

A ball whose mass is 0.2 kg is at rest on the ground. It is hit head-on by a second ball whose mass is 0.1 kg, and which is initially moving with a velocity of 3 m/s in the x direction. (Neglect all horizontal forces other than those that the two balls exert on each other.)

(a) Make a sketch and draw a dotted line around the system that you will consider. Show all momentum vectors.

(b) Is there a net force on the system?

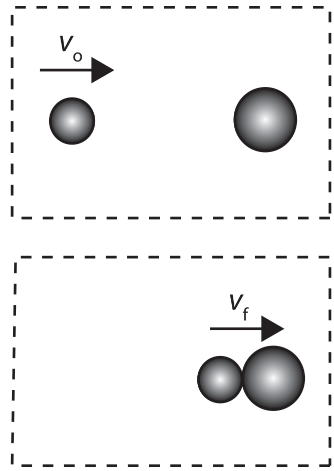
(c) After the collision the two balls stick to each other and move off together. What is their velocity right after the collision?

(d) In a different collision, starting as before, the balls do not stick to each other, and the ball that was initially at rest is observed afterward to move with a velocity of 1.2 m/s in the x direction. What is the velocity of the other ball after the collision?

(e) In still another collision, again with the same start, the ball that was originally at rest is observed after the collision to be moving with a velocity of 0.8 m/s in a direction at 45° to the x direction. Draw a diagram that shows all momentum vectors. From your diagram find (graphically and analytically) the momentum of the other ball after the collision. What is its velocity?

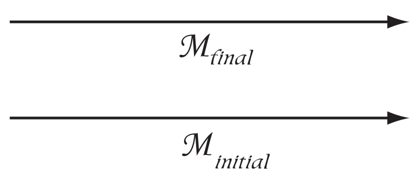
Ans.:

(a)



(b) No, the forces that the balls exert on each other are internal to the system. (There are vertical forces, but they add up to zero.)

(c) Before the collision the total momentum of the system is that of the moving ball, (0.1 kg)(3 m/s) = 0.3 kg m/s in the x direction.



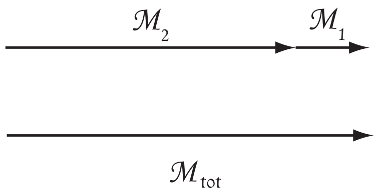
The momentum of the system consisting of the two particles is conserved, i.e., the momentum before the collision is equal to the momentum after the collision, still 0.3 kg m/s. (The diagram shows the initial momentum vector and the final momentum vector. They are equal in magnitude and direction.)

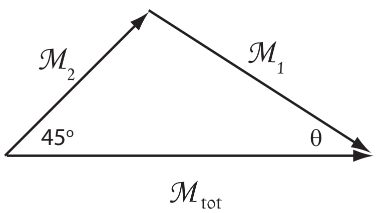
The mass of the system is the sum of the masses of the two particles, $0.2\,\text{kg} + 0.1\,\text{kg} = 0.3\,\text{kg}$. The velocity of the system of the two balls sticking together after the collision is therefore $\frac{0.3\,\text{kg m/s}}{0.3\,\text{kg}} = \frac{1\text{m}}{\text{s}}$ in the x direction.

(d) The initial total momentum is the same as in part (a), 0.3 kg m/s in the x direction. The total momentum is conserved, and is again the same after the collision.

The momentum of the 0.2 kg ball is (0.2 kg)(1.2 m/s) = 0.24 kg m/s in the x direction.

It is shown as $\mathcal{M}_2$ on the diagram. The momentum of the 0.1 kg ball is therefore 0.3 kg m/s − 0.24 kg m/s = 0.06 kg m/s. It is shown as $\mathcal{M}_1$ on the left-side diagram. Its velocity is $\frac{0.06\,\text{kg m/s}}{0.1\,\text{kg}} = \frac{0.6\text{m}}{\text{s}}$, in the x direction.





(e) The initial momentum is again the same, and since momentum is conserved, so is the momentum after the collision. But the directions are now not along the x direction. The diagram shows the momentum $\mathcal{M}_2$ after the collision of the 0.2 kg ball at 45° to the x-direction. Together with the momentum $\mathcal{M}_1$ of the 0.1 kg ball it must add up to the total momentum, which is again 0.3 kg m/s, both before and after the collision.

Graphically, from the momentum diagram, the momentum of the 0.1 kg ball ($\mathcal{M}_1$) is 0.22 kg m/s, at an angle of 31° to the x direction.

Analytically: $\mathcal{M}_2$ has two components, $\mathcal{M}_2 \cos 45 = (0.16)(.707) = 0.113$ kg m/s and $\mathcal{M}_2 \sin 45$, which has the same magnitude.

$\mathcal{M}_1$ has two components: $(M)_x = (M)_{\text{tot}} - \mathcal{M}_{2x} = 0.3 - 0.113 = 0.187$ and $\mathcal{M}_y$ whose magnitude is 0.113. The magnitude of $\mathcal{M}_1$ is $\sqrt{.187^2 + .113^2} = 0.219$.

The tangent of the angle θ is $\frac{.113}{.187}$, so that $\theta = 31.1°$.

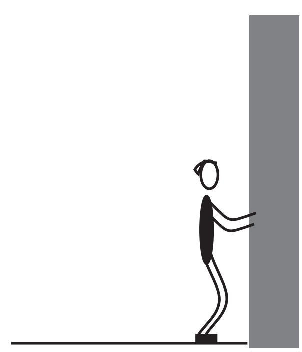
To find the velocity we divide by the mass to get 2.2 m/s.

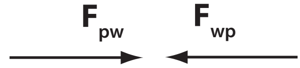
If the momentum of one of the two particles changes (still with no forces from outside the system), the other must change also, in the opposite direction, so as to keep their combined momentum constant. The rate at which the momentum of one changes must be just as large as the rate at which the momentum of the other changes, and in the opposite direction. But the rate at which the momentum of a particle changes is the force on it. We see that the force on one particle is equal in magnitude to the force on the other, and is in the opposite direction.

There is no force from the outside. The only forces are the ones inside the system that the two particles exert on each other. These are the force that particle one exerts on particle two ($\mathbf{F}_{12}$) and the force that particle two exerts on particle one ($\mathbf{F}_{21}$). These forces must be equal in magnitude and opposite in direction. $\mathbf{F}_{12} = -F_{21}$. This is Newton's third law.

But we knew this already! At least intuitively. Force is an interaction. The two objects that interact either attract or repel each other with forces that have the same magnitudes.

Newton's third law remains true regardless of any other forces. It says that all forces occur in pairs. If I push you, you experience a force. As I do that I experience a force on me from you, equal to my push and in the opposite direction. The same is true if I push against a wall. The wall pushes back on me with a force equal in magnitude and opposite in direction.

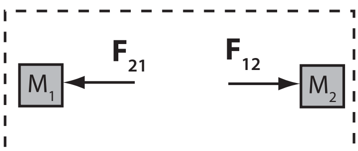




You may ask: "If all forces come in pairs that add up to zero, how can anything ever get going?" The two forces of a Newton's third law pair act on different objects! The net force on any one object is the sum of the forces that act on that one object, and this is what gives rise to its acceleration.

If there are two masses, M_1 and M_2, and you want to know how M_1 moves, you have to draw a force diagram for M_1. One of the forces on it will be $\mathbf{F}_{21}$, the force exerted on M_1 by M_2. There may be other forces on it. The vector sum of all the forces on M_1 determines what happens to M_1. Similarly, if you want to know what happens to M_2, and draw a force diagram for it, one of the forces (among others) will be $\mathbf{F}_{12}$.

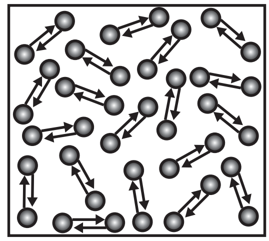
You may also want to draw a force diagram for the system composed of both M_1 and M_2.



This time the pair of forces $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$ are forces inside the system. They don't influence the motion of the system as a whole, only what happens inside it. They are *internal* forces that add up to zero.

Sometimes Newton's third law is described by saying that "for every action there is an equal and opposite reaction," but this is not a particularly helpful way to think about it. The words *action* and *reaction* are not defined, and they don't make it clear that the two forces act on different objects. Moreover a reaction is usually thought to come after the action, and this is not true here. The two forces that Newton's law refers to act simultaneously.

Next, think of a system or combined object consisting of many particles. Even the tiniest visible object consists of a huge number of microscopic pieces, its atoms and molecules. They exert forces on each other, each one on its neighbor, and to a lesser extent on those further away. Inside the atoms the nuclei and electrons also exert forces on each other. Each fragment of material is a seething mass of particles exerting internal forces on each other.



We can now see that there is a simple way to deal with these forces. After all, when we hold a ball in our hands we are not aware of all of the atoms and molecules that constitute it, nor of the forces between them. Because they come in pairs, each of which adds up to zero, we can just ignore them!

It is really important to be able to talk about a tennis ball or any other object without each time having to consider all the molecules (and their constituents) within it. We have always done that, and now we see that it is Newton's third law that allows us to do it.

Once we agree that the sum of the internal forces is zero, the net force acting on the system can be found without considering them, regardless of how many collisions the pieces make with each other, or how they separately change their velocity and their momentum. If there is no *external* force on the system then the total momentum of the system does not change. This is the *law of conservation of momentum*.

Look at just two particles or objects colliding with each other. Each will change its velocity, and its momentum. But the sum of the momenta (not the velocities!), i.e., the total momentum of the two combined, will be the same before the collision and after the collision.

This is the basis of our knowledge of all kinds of collisions, whether they are between cars or between the molecules of a gas.

EXAMPLE 19

The horse and the wagon

The way we use Newton's second and third laws is so important that we will go over it once more in this example.

A horse pulls on a wagon.

(a) Describe the force of interaction between the horse and the wagon.

(b) Describe the forces on the system containing both the horse and the wagon.

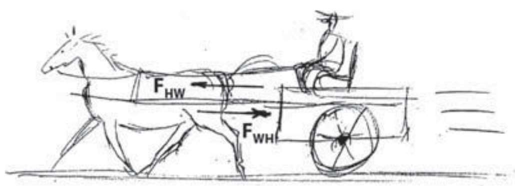
(c) Describe the forces on the system containing only the wagon.

(d) Describe the forces on the system containing only the horse.

Ans.:

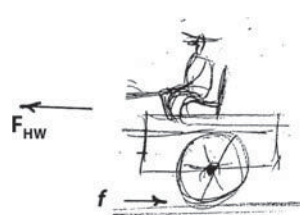
(a) Newton's second law deals with the motion of just one object. If we want to know the acceleration of that object we need to consider all the forces on that object. We have to be careful not to include forces that act on any other objects.

Newton's third law deals with pairs of forces, each acting on a different object. When a horse pulls on a wagon, the wagon pulls on the horse. Each of these two forces is an interaction between the horse and the wagon. The two forces have the same magnitude and are in opposite directions.

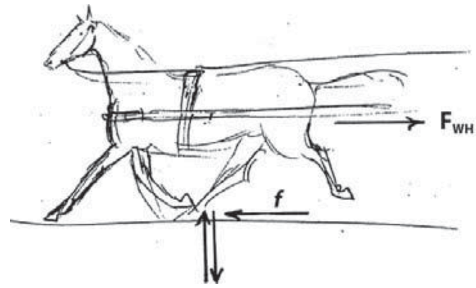


(b) For the system containing both the horse and the wagon the two interaction forces are internal forces. They add up to zero and do not contribute to the sum of the forces on the system or to the system's acceleration.

We can now list the external forces on the system. There are the usual two vertical forces, the weight and the normal force. The horizontal forces are the force of friction and air resistance.



(c) If we want to know the wagon's acceleration, we choose a system containing only the wagon. Just one of the two interaction forces comes into play, the force of the horse on the wagon, F_{HW}. There is also the force of friction on the wagon, and air resistance, in addition to the two vertical forces.



(d) If we want to talk about the horse, we choose a system containing only the horse. As we count the external forces on it, we need to include the force on the horse by the wagon, F_{WH}.

The force of friction acts to push the horse forward. We can see that by looking at what happens at the interaction between the horse's hoofs and the road. The horse pushes back on the road with a force F_{HR}. Paired to this force by Newton's third law is the force of the road on the horse, F_{RH}, forward.

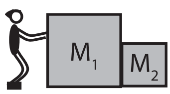
While we dealt with the horse and the wagon we ignored all of their internal structure. In other words we have used a model in which each is a particle. In the real world the horse is a wildly complicated object whose internal motions we can never know completely. Not only that, but we even have trouble deciding what the external forces are. The horse breathes, eats, eliminates, and interacts with its environment in ways that make it quite impossible to delineate its boundaries exactly. The model that we use does not include these forces.

We know that we can ignore its internal forces, the forces between its various parts and between each pair of molecules. Each pair of interaction forces is related by Newton's third law and adds up to zero.

Even the wagon is hardly the rigid and inert body that we might wish for to keep its description simple. It has wheels whose motion we haven't thought about. And on the microscopic scale it is a collection of constantly moving atoms and molecules. Again, a large part of the simplification that allows us to deal with the problem is that we can ignore the internal forces.

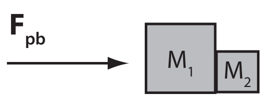
(The figure shows the internal forces between the horse's ears and its head.)

EXAMPLE 20



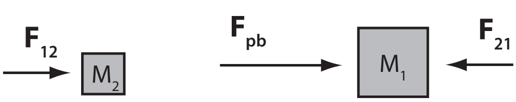
You are pushing two boxes along the floor with a force of 100 N. The box on which you are pushing has a mass of 12 kg and the other one has a mass of 8 kg. What is the force of the first box on the second box, and the force of the second box on the first? (Neglect friction and all other horizontal forces.)

Ans.:
We can find the acceleration most simply by first looking at the system that includes both boxes, with their combined mass of 20 kg. $\Sigma F = F_{pb} = Ma, a = \frac{F_{pb}}{M} = \frac{100\,\text{N}}{20\,\text{kg}} = 5\,\text{m/s}^2$.



Now we can apply Newton's second law to the lighter of the two boxes.

Since we are neglecting friction, the only horizontal force on the box is F_{12}. We already found the acceleration to be $5\,\text{m/s}^2$. $F_{12} = Ma = (8\,\text{kg})(5\,\text{m/s}^2) = 40\,\text{N}$ in the x direction. By Newton's third law this is also F_{21}.



We can check this result by using the force diagram for the heavier box.

$100 - F_{21} = (12\,\text{kg})(5\,\text{m/s}^2)$, or $F_{21} = 100 - 60 = 40\,\text{N}$, this time in the negative x direction.

EXAMPLE 21

An elevator whose mass is 250 kg hangs from a cable. Find the force of the cable on the elevator in the following situations. For each case draw a force diagram, write the relation between the forces using symbols only, and then substitute numbers for the symbols to find the force exerted by the cable.

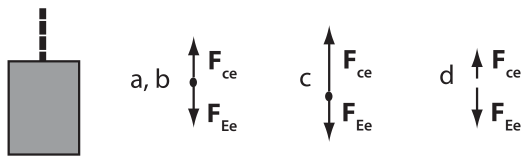
(a) The elevator is at rest.

(b) The elevator is moving down with a constant velocity of 2 m/s.

(c) The elevator is moving up, and accelerates upward with an acceleration of $1.5\,\text{m/s}^2$.

(d) The elevator is moving up and slowing down with an acceleration whose magnitude is $2\,\text{m/s}^2$.

Ans.:



(a) There are two forces on the elevator. Its weight, $F_{ee} = Mg$, and the force of the cable on the elevator, F_{ce}. Let up be positive. The net force is $F_{ce} - Mg$. Since the acceleration is zero, the net force is zero, and $F_{ce} - Mg = 0$. $F_{ce} = Mg = (250)(9.8) = 2450\,\text{N}$.

(b) There is no acceleration. The answers are the same as in part (a).

(c) Let up be positive again. The forces are F_{ce} and $-Mg$. Their sum is $F_{ce} - Mg$, and it is equal to Ma. $F_{ce} - Mg = Ma$. $F_{ce} = Mg + Ma = 2450 + (250)(1.5) = 2450 + 375 = 2.83 \times 10^3\,\text{N}$.

(d) Again the sum of the forces (up) is $F_{ce} - Mg = Ma$.

The acceleration is downward, hence negative, equal to $-2\,\text{m/s}^2$. $F_{ce} = 2450 + (250)(-2) = 2450 - 500 = 1.95 \times 10^3\,\text{N}$.

We can also choose down as positive, so that the acceleration is positive: The sum of the forces (down) is $Mg - F_{ce} = Ma$ and $F_{ce} = Mg - Ma = 2450 - (250)(2) = 1.95 \times 10^3\,\text{N}$, as before.

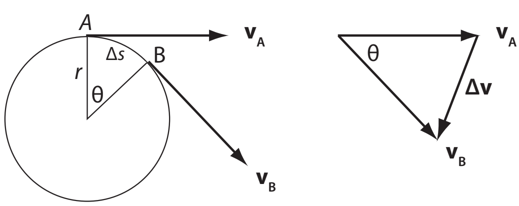
Later we will spend time with an example that goes beyond mechanics and has the most profound implications. Consider a roomful of air: billions of billions of molecules, most of them moving with speeds far greater than any vehicles on earth or in space, colliding with each other, recoiling, and exerting forces on each other. But if we take the whole amount of gas in the room as our system, all of the forces that the molecules exert on each other as they collide are internal forces, pairs of forces that are related by Newton's third law, and that add up to zero. Only the forces from outside the system, by the walls, as molecules bounce off them, need to be considered in order to deal with the system as a whole. We will see that this example opens the door to the understanding of the relation between mechanics and the phenomena of heat and temperature.

4.4 One more motion that is everywhere: rotation

Uniform circular motion

When a particle does not move in a straight line, even if its speed does not change, it has an acceleration and so there must be a net force on it.

We will look at a particle moving with constant speed in a circle to see what we can say about its acceleration and about the force that must act on it to keep it moving in its circle. This kind of motion is called *uniform circular motion*, where "uniform" refers to the constant, or *uniform* speed.



The left-hand part of the figure shows a particle and the circular path along which it moves. As the particle moves from A to B, it moves through a distance Δs along the circumference, and the radius of the circle turns through an angle θ. This angle, in *radians*, is equal to $\frac{\Delta s}{r}$. (For motion all around the circle, through 360°, the number of radians is equal to the circumference of the circle divided by its radius, or $\frac{2\pi r}{r}$, so that 2π radians equal 360°.)

The velocity, as a vector quantity, has both magnitude and direction, and therefore changes as the particle moves. At point A the velocity is $\mathbf{v_A}$ and at point B it is $\mathbf{v_B}$. The change in the velocity between these two points is $\mathbf{v_B} - \mathbf{v_A}$, which we call $\Delta\mathbf{v}$. (Look at it as $-\mathbf{v_A} + \mathbf{v_B}$, i.e., go backward along $\mathbf{v_A}$ and then forward along $\mathbf{v_B}$. The vector from the beginning of this path to the end is $\Delta\mathbf{v}$.)

The right-hand part of the figure is a vector diagram that shows the three vectors $\mathbf{v_A}$, $\mathbf{v_B}$, and $\Delta\mathbf{v}$. The velocity vector remains at right angles to the radius, and as the radius turns through the angle θ, so does the velocity. The length (or magnitude) of the velocity vector is the speed, v, and for the motion that we are considering it remains constant.

The angle ($\theta = \frac{\Delta s}{r}$ radians) between the radii to A and B is the same as the angle between the two velocities. The vector diagram shows that it is also approximately equal to $\frac{\Delta v}{v}$, and becomes closer to it as the angle gets smaller.

Hence $\theta = \frac{\Delta s}{r}$ and $\theta = \frac{\Delta v}{v}$, so that $\frac{\Delta s}{r} = \frac{\Delta v}{v}$. Dividing by Δt (the time interval during which the particle moves from A to B), we get $(\frac{1}{r})(\frac{\Delta s}{\Delta t}) = (\frac{1}{v})(\frac{\Delta v}{\Delta t})$. This is the same as $(\frac{1}{r})(v) = (\frac{1}{v})(a)$. We can now solve for a to get $a = \frac{v^2}{r}$.

It is not surprising that a is proportional to v^2. We know that a is close to $\frac{\Delta v}{\Delta t}$ (and gets closer to that as Δt and Δv get smaller). With larger speed (v), the change in v (called Δv) increases. In addition, as v gets larger, Δt (for the same displacement) decreases. The two changes together cause $\frac{\Delta v}{\Delta t}$, and hence a, to be proportional to v^2.

This acceleration is called the *centripetal* acceleration. (It means "pointing to the center.") When the angle θ gets smaller, $\Delta\mathbf{v}$ becomes perpendicular to $\mathbf{v_A}$ (or $\mathbf{v_B}$), and is directed toward the center of the motion. A net force with magnitude $\frac{mv^2}{r}$ (called the *centripetal force*) is required to keep the particle moving in the circle with radius r with speed v.

We know that a net force is necessary for an object to move in a circle. Without one it would move with constant velocity, i.e., with constant speed in a straight line, in accord with Newton's first law of motion. Now we see that for motion in a circle with constant speed the force has to point toward the center of the motion, and have magnitude $\frac{mv^2}{r}$. There needs to be a centripetal (or center-pointing) force. This is not a new or additional force. Whatever the forces on the object are, if the object is to move in a circle with constant speed (in *uniform circular motion*) their sum has to point to the center and have magnitude $\frac{mv^2}{r}$.

This is true regardless of the nature of the interaction that leads to the circular motion. For the moon going around the earth the centripetal force is the gravitational force. That's also true for the earth going around the sun. For a ball being swung in a circle at the end of a rope it is the force of the tension of the rope. For a particle in a cyclotron it is the magnetic force. In each of these cases there is an object moving in a circle. In each case the magnitude of the net force is equal to $\frac{mv^2}{r}$, regardless of the nature of the force.

EXAMPLE 22

Go to the PhET website (http://phet.colorado.edu) and open the simulation *Ladybug Revolution*.

Choose the "Intro" tab. Check "Show velocity vector" and "Show Acceleratin Vector."

(i) Set the angular velocity to zero.
Drag the ladybug with the mouse. Observe the vectors **v** and **a**.

(ii) Set the disk in motion with the slider at the bottom. Set the angular velocity to about 150 degrees/s. Observe the two vectors.
Observe the effect on the two vectors of changing the angular velocity and the ladybug position.

(iii) Choose the "Rotation" tab.
Check "Show Ladybug Graph" "Show Platform Graph," and (θ, ω, v). Set ω to about 4 rad/s. The graphs include the values of ω and v. Calculate r from them.
Check "ruler" and measure the radius, r, at which the ladybug sits. Compare this to the result of your calculation.
What are a and α?
Check the graphs that contain a and α, and compare your results with those on the graphs.

(iv) Check "Show ladybug graph" and check the graph θ, ω, x, and y and "Show X-Position" and "Show Y-Position." Reset all, and set θ to zero. Set the platform in motion again and observe the graphs of x and y. These are the x and y components of the radius vector **r**, i.e., $x = r\cos\theta$ and $y = r\sin\theta$. Observe that they are similar except for being displaced with respect to each other along the time axis (or *out of phase*). At $\theta = 0$, $\sin\theta = 0$ and $\cos\theta = 1$. Both components are said to vary *sinusoidally*. (Note that you can change the horizontal and vertical scales of the graphs with the buttons on the right.) Look for the time of a complete revolution on the graph and compare it to the value derived from ω. Use the step function to compare and explore what you see on the platform and on the graph for x and y.

The sinusoidal variation is typical of oscillations in many parts of physics. We will see it for masses on springs, sound, and other waves, and for atomic and molecular vibrations. Motion in which the position varies sinusoidally is called *simple harmonic motion*.

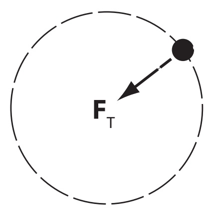
EXAMPLE 23

Anja swings a ball at the end of a string so that it moves in a horizontal circle. The ball's mass is 0.3 kg. The radius of the circle is 1.2 m. The ball travels around the circle in one second, i.e., it makes one revolution per second. (Neglect the vertical forces.)

(a) What is the magnitude of the force required to make the ball move in the circle?

(b) What force acts on the ball to keep it moving in the circular path?

(c) What pulls on Anja's hand, and how hard does it pull?



Ans.:

(a) The circumference of the circle is $2\pi r = (2)(\pi)(1.2) = 7.54$ m. The speed of the ball is therefore 7.54 m/s. The force that is required (the centripetal force) is $\frac{Mv^2}{r} = \frac{(0.3)(7.54)^2}{1.2} = 14.2$ N.

(b) The force on the ball is that of the string. The force exerted by the string is its tension of 14.2N.

(c) The string pulls on the ball toward the center of the motion, i.e., toward Anja's hand. At its other end it pulls on Anja's hand toward the ball. As long as we neglect the mass of the string, the two forces have the same magnitude.

EXAMPLE 24

The astronaut Sally Ride (whose mass is 60 kg) is in her spaceship in a circular orbit 100 km above the surface of the earth.

(a) Use proportional reasoning to find her weight in orbit.

(b) What is the interaction that keeps her in orbit?

(c) How large is the force on her?

(d) What is the time, T, for a complete orbit?

Ans.:

(a) Her weight on earth is $Mg_e = (60)(9.8) = 588\,\text{N}$, where g_e is the value of g on earth. Since $Mg_o = G\frac{MM_e}{R^2}$, where g_o is the value of g in orbit, g is proportional to $\frac{1}{R^2}$. Hence $\frac{g_e}{g_o} = \left(\frac{R}{R_e}\right)^2 = \left(\frac{6470}{6370}\right)^2 = 1.031$. In other words, her weight on earth is 3.1% greater than it is in this orbit. Her weight in orbit is 570 N.

(b) The gravitational interaction keeps her in orbit.

(c) The force on her is the gravitational force, and it is 569 N.

The spaceship is in orbit with her. There is no force between her and the spaceship. This gives rise to the (incorrect) description of her as being "weightless." Sometimes the interaction between her and the spaceship is called the "apparent weight," which is then zero while she is in orbit. According to our definition the weight is equal to the gravitational force on her and is not zero.

(d) We can find the time T for a complete revolution from the relation for the centripetal force, which here is the gravitational force, so that $G\frac{MM_e}{R^2} = M\frac{v^2}{R}$.

We start by canceling M and dividing each side by R, to get $G\frac{M_e}{R^3} = (\frac{v}{R})^2$. Then we can use the fact that in the time T she and the spaceship go a distance $2\pi R$, so that $2\pi R = vT$, or $\frac{v}{R} = \frac{2\pi}{T}$. We can substitute $\frac{2\pi}{T}$ for $\frac{v}{R}$ to get $\frac{GM_e}{R^3} = (\frac{2\pi}{T})^2$, which we can turn around to get $T^2 = \frac{4\pi^2 R^3}{GM_e}$.

All that's left is to put numbers in: $M_e = 5.98 \times 10^{24}\,\text{kg}$ and $G = 6.67 \times 10^{-11}\,\frac{\text{Nm}^2}{\text{kg}^2}$. $R_e = 6.37 \times 10^6\,\text{m}$, so that $R = R_e + 0.1 \times 10^6\,\text{m} = 6.47 \times 10^6\,\text{m}$. Putting these numbers in leads to $T = 5.18 \times 10^3\,\text{s}$ or 86 min.

If we had used R_e instead of R the result would have been 84 min. We see that while the height above the earth, of 100 km, seems quite large to us, it changes the distance from the center of the earth by only about $1\frac{1}{2}$%. That's why satellites that travel reasonably close to the earth all take about $1\frac{1}{2}$ h for one orbit.

The same considerations can be used for the time of a complete orbit of a planet around the sun. The fact that $T^2 \propto R^3$ for the planets was discovered by Kepler, and is known as Kepler's third law.

The orbits of the planets are actually elliptical, but sufficiently close to being circular that the approximation of considering them to be circular is quite close.

Angular momentum and torque

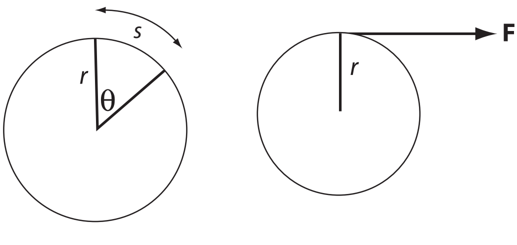
The momentum of an object or system describes its linear motion. The momentum remains constant unless an external force causes it to change.

There is an analogous quantity called the *angular momentum*, for rotational motion. It also remains constant, unless an external influence (a *torque*) causes it to change. The earth keeps rotating around the sun, and the moon around the earth, and in addition both keep spinning, in accord with the *law of conservation of angular momentum*. Angular momentum is a fundamental property of electrons, protons, and neutrons, and hence of the nuclei, atoms, and molecules of which they are the parts. The angular momentum of an atom depends primarily on its electrons. It plays a major role in determining the atom's properties.

As you can see, rotational motion is in some ways more basic than linear motion. To describe it in detail we will have to introduce some new terms, but each of them will be analogous to the corresponding term for linear motion.

Look at a single particle with mass m moving with constant speed, v, in a circle whose radius is r. Let's start with *angular displacement*. This is the angle θ through which the particle travels. We will measure it in *radians*, so that $\theta = \frac{s}{r}$, where s is the distance along the circumference.

Now we can define the *angular velocity* (ω, Greek *omega*) as the rate of change with time of



the angular displacement. $\omega = \frac{d\theta}{dt}$, which is also equal to $\frac{1}{r}\frac{ds}{dt}$, or $\frac{v}{r}$.

The *angular acceleration* (α, Greek *alpha*) follows, as the rate of change with time of the angular velocity. $\alpha = \frac{d\omega}{dt}$, which is also equal to $\frac{1}{r}\frac{dv}{dt}$, or to $\frac{a}{r}$.

All the relations between x, v, and a have their counterparts in relations between θ, ω, and α, and can be used in the same way. For instance, as the relation corresponding to $v = v_0 + at$ we can write $\omega = \omega_0 + \alpha t$ to show how the angular velocity varies with time when there is a constant angular acceleration.

So far we have talked about a single particle, but the same considerations apply to an extended rotating object, as for example, a wheel.

EXAMPLE 25

The drive shaft of a car engine is rotating with an angular velocity of 100 rpm (revolutions per minute). It decelerates with a constant angular acceleration of 2 rad/s^2 (radians per second squared). How long does it take for the engine to come to rest?

Ans.:

First we have to see that all the quantities that we need to use are in SI units. A *revolution* is 2π *radians*, so that 100 rpm $= (100)(2\pi)$ rad/min or $\frac{100}{60}2\pi$ rad/s. Now we can use $\omega = \omega_0 + \alpha t$, where the final angular velocity, ω, is zero, the initial angular velocity, ω_0, is $\frac{200\pi}{60}$ rad/s, and $\alpha = -2$ rad/s^2. (α has the opposite sign from ω because the engine is slowing down.) We turn the relation around to read $t = \frac{\omega - \omega_0}{\alpha}$ and substitute the numbers to get $t = \frac{100\pi}{60} = 5.24$ s.

What takes the place of the force? If we apply a force to our particle, tangential to the circle in which it moves, it matters not only how large the force is, but also where it is applied. It will have a greater effect on the rotation if it is applied further from the center. We therefore define the torque (τ, Greek *tau*) as Fr, where F is the tangential force and r is the radius at which it is applied.

We see that $\tau = Fr = mar = m\frac{a}{r}r^2$, or $\tau = mr^2\alpha$. This has the same form as Newton's second law, $F = Ma$, with torque taking the place of force and angular acceleration taking the place of the linear acceleration. The quantity mr^2 takes the place of the mass, and is given the somewhat cumbersome name *moment of inertia* with the symbol I. The torque produces an angular acceleration. The size of the angular acceleration depends on the moment of inertia. $\tau = I\alpha$.

We can extend the description from that of a particle to that of an extended object. The torque and angular acceleration are the same as before. The moment of inertia is the sum of the moments of inertia for all the pieces of the object. We can write it as $I = \Sigma mr^2$, where m is the mass of one piece, at a distance r from the axis about which the object rotates, and the symbol Σ stands for "sum." The moment of inertia is the quantity that for the rotational motion of an extended object is analogous to the mass for linear motion. It depends on how the mass is distributed in the object.

If the mass is distributed in discrete pieces, we can simply add mr^2 for each to find Σmr^2. It may also be distributed continuously, as in a disk or wheel, or sphere. It is then often possible to find a simple expression for the moment of inertia. For example, the moment of inertia of a disk of mass M with radius R, rotating about an axis through its center, is $\frac{1}{2}MR^2$.

The linear momentum is Mv. In analogy we can now define the angular momentum as $I\omega$, the product of the moment of inertia and the angular velocity. For a single particle moving in a circle it is equal to $(mr^2)(\frac{v}{r})$, or mvr.

(There is also rotational kinetic energy, analogous to the linear kinetic energy $\frac{1}{2}Mv^2$. It is equal to $\frac{1}{2}I\omega^2$. More about that in Chapter 6.)

Go to the PhET website and open the simulation *Torque*.

Go to the tab "Moment of Inertia." Explore the relation between torque, moment of inertia, and angular acceleration. Set a torque and observe the acceleration of the platform. Look at the values of these three quantities on the graph and check that their relation is what you expect from the rotational analog of Newton's second law of motion.

With the tab Torque you can explore the relation between the force and the torque, and with the tab Angular Momentum the relation between angular velocity, moment of inertia, and angular momentum.

The moment of inertia of a uniform solid disk is $\frac{1}{2}MR^2$. Explore the relation between M, R, and I by varying these quantities with the sliders.

EXAMPLE 26

A skater has a moment of inertia of 25 kg m^2 when she stretches out her arms. She starts out with an angular velocity of 3 rpm. She then draws her arms in and her moment of inertia is reduced to 10 kg m^2. What is her angular velocity now? (Assume that the external torque is zero.)

Ans.:
Angular momentum is conserved, so that $I_1\omega_1 = I_2\omega_2$, or $\frac{\omega_2}{\omega_1} = \frac{I_1}{I_2} = \frac{25}{10} = 2.5$. In the expression $\frac{\omega_2}{\omega_1}$ it doesn't matter what the units are as long as they are the same in the numerator and the denominator. We can therefore leave them as rpm to find that $\omega_2 = 2.5\omega_1 = 7.5$ rpm.

The table shows the various linear quantities and their rotational analogs.

Linear	*Rotational*
x	$\theta = \frac{s}{r}$
v	$\omega = \frac{v}{r}$
a	$\alpha = \frac{a}{r}$
F	$\tau = Fr$
M	$I = \Sigma mr^2$
Mv	$I\omega$
$\frac{1}{2}Mv^2$	$\frac{1}{2}I\omega^2$
$F = Ma$	$\tau = I\alpha$

The angular momentum of particles

The proton, neutron, and electron each has angular momentum at all times, called their *spin*. This spin angular momentum is also called their *intrinsic* angular momentum, to indicate that it is always there, regardless of any other motion. If an electron is part of an atom it may also have *orbital* angular momentum. Similarly the protons and neutrons move in a nucleus and may have orbital angular momentum in addition to their ever-present intrinsic spin angular momentum. The amounts of the angular momentum of an atom and a nucleus are among their most important properties.

We have to remember that the picture of the particles as little balls spinning about an axis is not correct. This is also true about the picture of an atom, as first envisioned by Bohr, with electrons in orbit, similar to planets about the sun. The picture of a spinning particle, or of an atom with electrons in orbit, may be a simple visualization. But a model in which ordinary (classical) mechanics is used for them does not lead to the correct (observed) results.

We will return to these questions later.

EXAMPLE 27

In its lowest energy state (the ground state) the helium atom has an electronic angular momentum of zero. Explain.

Ans.:
Although each of the two electrons in the helium atom has a spin angular momentum, the spins are in opposite directions. A *mechanical* picture is inappropriate for atoms, but in the ground state the two angular momenta nevertheless add up to zero. Each electron also has an orbital angular momentum. They are also in opposite directions.

As an aside we can mention the nuclear spin angular momentum. Most commonly, the nucleus of helium is an *alpha* particle, which consists of two protons and two neutrons. Again the total spin is zero. In 1.4×10^{-6}% of the atoms, however, the nucleus consists of two protons and only one neutron, to form the isotope ^{3}He, with a net spin angular momentum.

4.5 Summary

Everything we do and everything that happens around us involves forces. The basic feature of forces is that they give rise to acceleration. This is embodied in the relation $\mathbf{\Sigma F} = M\boldsymbol{a}$, Newton's second law of motion. Here $\mathbf{\Sigma F}$ is the symbol for the *vector sum* of all the forces acting on an object or *system*, or the *net force*, and $\boldsymbol{a}$ is the acceleration. M is the object's mass. It determines the magnitude of the acceleration.

In the SI system of units M is measured in kilograms (kg) and a in $\frac{\text{m}}{\text{s}^2}$. If these units are used, the unit for force, equal to $\frac{\text{kg m}}{\text{s}^2}$, is the *newton* (N).

Here are some special cases: When **ΣF** is zero (when the forces on an object add up to zero) the acceleration is zero. The object may be at rest, but it can also be in motion with constant velocity. (There is a nonzero acceleration only when the velocity changes.)

When **ΣF** is constant, the acceleration is also constant. In that case the relations for constant a apply: $v = v_0 + at$, $x = v_0 t + \frac{1}{2}at^2$, $v^2 = v_0^2 + 2ax$.

When all the forces are along the same line, they can be added by using only positive and negative numbers ("algebraically"). If they are not along one line they have to be added as *vectors*. This can be done in two ways. One is to draw the vectors end to end, one arrow following the other. Their sum is then represented by the single vector from the beginning (the "tail") of the first to the end (the arrowhead) of the last.

The other method of adding vectors is to use a coordinate (x–y) system and to decompose each vector into its x component and its y component. All the x components are then added algebraically to give ΣF_x and the y components are added to give ΣF_y. These two quantities are the components of the single vector **ΣF**, the vector sum of all the forces.

To find **ΣF** we draw a *force diagram*. This is a figure that shows vectors representing each of the forces (and nothing else!). The force diagram helps us to visualize the magnitude and direction of all the forces acting on an object.

The *momentum* of an object or system is its mass times its velocity. Since it is proportional to the velocity, it is also a vector quantity. If there is no net force on a system or object, its momentum is constant. This is the *law of conservation of momentum.*

When two objects (A and B) interact, the force of A on B has the same magnitude as the force of B on A and is in the opposite direction. This is *Newton's third law of motion.*

Angular motion is represented by the angular displacement, θ, the angular velocity, ω, equal to $\frac{d\theta}{dt}$, and the angular acceleration, α, equal to $\frac{d\omega}{dt}$. In analogy to linear motion the angular velocity is the rate of change with time of the angular displacement.

"Uniform circular motion" is motion of a particle in a circle with constant speed. It is not constant velocity, because the direction of the velocity keeps changing. The acceleration also has constant magnitude and changing direction. Its direction is toward the center of the motion. It is called the "centripetal" ("toward the center") acceleration. Its magnitude is related to the speed and radius by the relation $a = \frac{v^2}{r}$.

In order for a particle to move in a circle with uniform circular motion there must be a net force toward the center (the "centripetal force"), whose magnitude is equal to $\frac{mv^2}{r}$.

Just as force gives rise to acceleration, *torque* gives rise to angular acceleration. And just as the mass of an object determines how large the acceleration of an object is when a net force acts on it, the *moment of inertia* determines how large the angular acceleration of a rotating object is when a net torque acts on it. The moment of inertia is a measure of how the mass is distributed.

Newton's second law also holds for rotational motion. Analogous to $\mathbf{\Sigma F} = M\boldsymbol{a}$ there is the relation for angular motion $\Sigma\tau = I\alpha$, where $\Sigma\tau$ is the sum of all the torques, I is the moment of inertia, and α is the angular acceleration.

The *angular momentum* (analogous to the linear momentum Mv) is $I\omega$. For a system with no net external torque on it, the angular momentum remains constant. This is the *law of conservation of angular momentum.*

To find the moment of inertia of an object with respect to an axis, we have to imagine cutting it up into pieces of mass m, each a distance r from the axis, and adding the values of mr^2 for each piece to get the sum $I = \Sigma mr^2$.

There is also kinetic energy associated with rotating objects. The *angular kinetic energy* is $\frac{1}{2}I\omega^2$, analogous to the linear kinetic energy, $\frac{1}{2}mv^2$. (See Chapter 6.)

4.6 Review activities and problems

Guided review

1. A book is at rest on a table. What are the forces on it?

2. A car whose mass is 2000 kg accelerates with $a = 2.5\,\text{m/s}^2$. What is the net force on it?

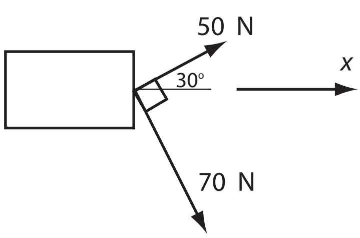
3. Maja's weight is 120 pounds. What is her mass in kg and her weight in N?

4. A box is being pulled forward horizontally along a table by a force of 10 N. The force of friction is 1.5 N.

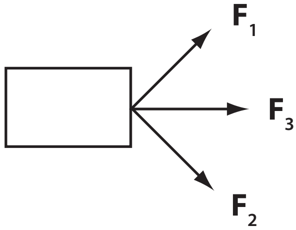
(a) Make a diagram that shows all four forces on the book.

(b) Write the mathematical statement of Newton's second law with these forces.

5. A wagon whose mass is 12 kg is being pulled by two horizontal forces. The first is 50 N at 30° to the x-axis. The second is 70 N at right angles to the first. What is the magnitude of the wagon's acceleration?



6. F_1 and F_2 are two horizontal forces of 60 N each, acting on a wagon. They are at 45° to the x-axis and at right angles to each other. A third force, F_3, also 60 N, acts along the positive x-axis. What are the magnitude and direction of the net force on the wagon?



7. For the three forces of the previous question, find $\mathbf{F}_1 + \mathbf{F}_3 - \mathbf{F}_2$.

8. A box whose mass is 23 kg is being pulled horizontally along the floor by a rope with a force of 120 N. The force of friction is 32 N.

(a) Draw a force diagram, indicating each of the forces on it with a symbol that has the appropriate two subscripts.

(b) Find the net force on the box, and its acceleration.

9. Go to the PhET website and open the simulation *Forces and Motion*.

Choose an object other than the crate. Answer the same questions as in the example. (For the first three parts choose a new appropriate applied force.)

10. Repeat Question 8 with the rope at an angle of 20° to the horizontal. Assume that the coefficient of friction (the ratio of the force of friction to the normal force) remains the same.

11. Go to the PhET website and open the simulation *The Ramp: Forces and Motion*.

(a) Choose the Intro tab.

Choose the file cabinet. Find the largest angle at which it does not slide down. What are the force of friction and the normal force at this angle? Check that their ratio is the marked coefficient of friction.

(b) Set the ramp angle at 10°. What is the force of friction now? Why is it not equal to μN?

(c) Go to *Force Graphs*. Put the file cabinet on the ramp, set the angle at 20°, and apply a force of 700 N. Push to go. Calculate all the forces. Compare your numbers to the ones on the screen.

12. A force in the $x-y$ plane whose magnitude is 12 N makes an angle of 125° with the $(+x)$ direction. What are its x and y components?

13. A car whose mass is 1800 kg is at rest on a road that is inclined at an angle of 8° to the horizontal. Draw a force diagram and find the magnitude and direction of each of the forces on the car.

14. A toboggan slides down a hill inclined at 20° with an acceleration of $1.5\,\text{m/s}^2$. Draw a force diagram showing all the forces. What other information must you know to calculate the magnitude of each of the forces?

15. A stone falls vertically from a roof straight down to the ground. (Air resistance may be

neglected.) Draw a force diagram for the time that it is in the air. What is its acceleration?

16. A rifle bullet is shot out of a gun that points at an angle of 30° to the horizontal. For the moment that the bullet is at its highest point:

(a) Draw a force diagram.

(b) What is the direction of is velocity?

(c) What are the magnitude and direction of its acceleration?

17. Two railroad cars, each with a mass of 10^4 kg, are connected to each other. They are being pulled horizontally by a locomotive with an acceleration of 0.1 m/s^2. (Neglect friction.)

(a) Draw a force diagram for each of the two cars and for the system containing both.

(b) Find the magnitude of each of the forces on your force diagrams.

18. A car whose mass is 2000 kg moves with a velocity of 30 m/s. It hits a stationary car that has the same mass. (Ignore all horizontal forces except for the ones that the two cars exert on each other.)

(a) The two cars stick to each other after the collision. What is their velocity after the collision?

(b) In a different collision, with the same start, the cars do not stick to each other. After the collision the car that was originally at rest is observed to move with a velocity of 10 m/s at an angle of 35° with the original motion of the other car. Draw vector diagrams that show the momentum vectors before and after the collision. Find the momentum and velocity after the collision of the car that was originally moving.

19. Two railroad cars, each with a mass of 2×10^4 kg, are being pushed by a locomotive with a force of 1.5×10^5 N. (Neglect friction.)

(a) Draw force diagrams for each of the two cars and for the system containing both.

(b) Find the magnitudes of each of the forces on your force diagrams.

20. You are pushing three equal boxes (8 kg each) along the floor with a force of 200 N. (Neglect friction.) What are the two horizontal forces on the middle box?

21. A boulder whose mass is 450 kg is being lifted vertically by a chain with an acceleration of 0.2 m/s^2.

(a) Draw a force diagram for the boulder.

(b) Find the magnitudes of each of the forces on your diagram.

(c) The boulder is now moving down and slowing down with an acceleration whose magnitude is 0.8 m/s^2. What is the direction of the acceleration? Draw a force diagram and find each of the forces on it.

22. Go to the PhET website (http://phet.colorado.edu) and open the simulation *Ladybug Revolution*.

Choose the "Rotation" tab.

Check "radians," "Show Ladybug Graph," "Show Platform Graph" "Show Acceleration," and uncheck the others.

Put the ladybug in the green zone and measure its radius with the ruler.

(a) Set the disk in motion with an angular velocity of about 3 rad/s.

Calculate the centripetal acceleration.

Compare your value of the centripetal acceleration with that shown on the graph of $a(t)$.

(b) Check "X-Acceleration" and "Y-Acceleration." Describe and explain the difference between the graphs of these two quantities.

23. A car travels in a circle whose radius is 20 m, with a speed of 25 m/s.

(a) What is the magnitude of the car's acceleration?

(b) What is the nature (origin) of the horizontal force on the car? (What is exerting the force?)

24. (a) At what height above the surface of the earth is an astronaut's weight half of what it is on earth?

(b) What is the time of an orbit of a satellite around the moon close to the moon's surface? (What quantities do you have to look up to be able to answer this question?)

25. A flywheel speeds up from rest, with an angular acceleration of 1 rad/s^2. What is its angular velocity after one minute in rad/s and in rpm (revolutions per minute)?

26. A disk with a moment of inertia of 3 kg m^2 spins with an angular velocity of 8 rad/s. A second disk, with a moment of inertia of 2 kg m^2 is initially at rest, but is free to rotate on the same shaft. It is now pressed against the first disk, as in a clutch. What is the angular velocity of the

combined system of the two disks in contact with each other?

27. The spin angular momentum of the ground state of the lithium atom ($Z = 3$) and that of the sodium atom ($Z = 11$) is each equal to that of a single electron. Explain why this is so.

Problems and reasoning skill building

1. It is sometimes said that when you travel around a circle in a car there is a "centrifugal" force that pushes you outward. What is it that really happens?

2. A roller coaster has a part with a vertical circular loop where, at its top, the car travels upside down.

(a) Draw a force diagram of the forces on the car at the top of the loop.

(b) What must be true about the speed so that the car and passenger do not fall out?

3. A skier comes down a 10° slope.

(a) Draw a force diagram of the skier as he comes down the slope. Do not neglect friction.

(b) What measurements could you make to determine if there is any friction between the skier and the slope? If there is friction describe how you could measure the force of friction.

4. Maya pulls three of her children on a sled. You are standing nearby with a meterstick and a stopwatch.

(a) Draw a force diagram of the sled with the children on it (do not neglect friction)

(i) when she pulls horizontally

(ii) when she pulls at an angle of 30° with the horizontal

(b) What is the relation between the forces when she pulls horizontally

(i) when the sled moves with constant velocity

(ii) when the sled accelerates

5. You pull on a rope so that it exerts a 30 N force on a 20 kg sled that moves on a level frictionless icy surface to the right. The force is directed at an angle of 30° above the horizontal.

(a) Draw a force diagram of the sled.

(b) A student constructed the equations below to describe the forces and motion of the sled, using SI units. Are these descriptions consistent with the word description? If not, correct the mathematical descriptions. (Let the direction to the right be the positive x direction and let up be the positive y direction.)

$$x : +30 = 20a_x$$
$$y : F_{\text{normal}} - (20)(9.8) - 15$$

(c) At time T_1 the sled is moving at a velocity of 1.2 m/s to the right. How fast will it be moving two seconds later?

6. (a) A 4 kg block is pulled from rest along a horizontal surface by a rope with a force of 10 N. It experiences a constant frictional force of 2 N. How far has the block moved in the first three seconds?

(b) If the rope in part (a) breaks at $t = 3$ s, what will the block do: stop immediately, slow to a stop, or continue moving at a constant speed? Explain your answer.

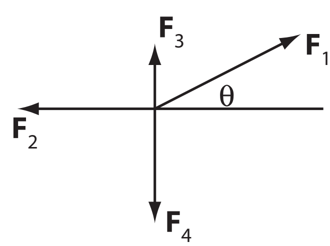
7. A 15 N thrust is exerted on a 0.5 kg rocket for a time interval of 8 s as it moves straight up. (Neglect all forces except this thrust and the force of gravity.) The rocket then continues to move upward as its speed reduces steadily to zero.

(a) Draw force diagrams of the rocket (i) for $0 < t < 8$ s and (ii) for $t > 8$ s.

(b) Describe the motion of the rocket by drawing graphs of v vs. t and a vs. t. You may assume that the rocket does not go so high that g changes significantly.

(c) What is the maximum height that the rocket attains?

8. The figure shows the force diagram of a 0.9 kg object moving horizontally. $F_1 = 10$ N, $F_3 = 4$ N, and $\theta = 30°$.

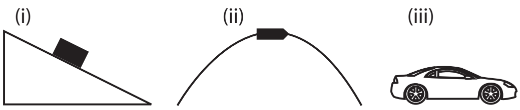


(a) Determine the magnitude of F_2 and F_4 if the object is moving horizontally with constant velocity.

(b) Find the acceleration if F_2 is 4 N.

(c) Describe a physical situation that could be represented by this diagram. What is the source of each of the three forces?

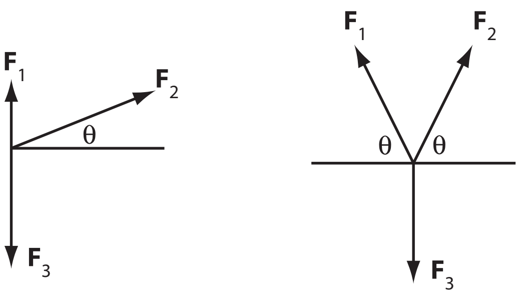
9. Three different motions are described in the diagram: (i) shows a block accelerating down an incline, (ii) is a projectile at the top of its trajectory (neglect air resistance) and (iii) is a car moving to the right and slowing down with constant acceleration.



(a) For each of the three motions draw a vector **v** that shows the direction of the velocity and a vector $\mathbf{F_{net}}$ that describes the net force.

(b) For each of the three cases specify a coordinate system and sketch a graph that describes the acceleration as a function of time.

10. For each of the force diagrams, find $\mathbf{F_{net}}$ in terms of the magnitudes F_1, F_2, F_3 and the angle θ.



11. For each of the following situations:

(a) Draw a pictorial representation including a symbolic representation of all the information you are given and the assumptions that you make.

(b) Draw a force diagram.

(c) Find ΣF_x and ΣF_y.

(i) A hockey puck is pushed horizontally with a force $F_{\text{stick on puck}}$ across the ice. (Neglect friction.)

(ii) A ball falls through the air.

(iii) A ball has been thrown upward and is now moving up.

12. A hockey puck of mass M is given a horizontal push of magnitude $F_{\text{stick on puck}}$.

(a) At what rate does its speed change?

(b) Use reasonable values to get numerical results. Is the answer reasonable?

13. A crate of unknown mass is pulled upward by a rope. The tension in the rope is 22 N and the crate accelerates upward at a rate of 1.2 m/s^2.

Determine the mass of the crate.

14. An elevator of unknown mass moves upward with a constant velocity v_0. The tension in the cable pulling the elevator is F_{re} ($F_{\text{rope on elevator}}$).

(a) Draw a pictorial representation including a symbolic representation of all the information you are given and the assumptions that you make.

(b) Draw a force diagram.

(c) Write a mathematical descriptions that allows you to determine the mass of the elevator.

(d) $v_0 = 3$ m/s up and $F_{\text{re}} = 5000$ N. Determine the mass. Is your answer reasonable?

15. A child with a mass of 40 kg stands on a spring scale inside an elevator. For each of the scenarios below, draw a force diagram of the forces acting on the child and calculate the reading of the scale in newtons.

(a) The elevator is at rest.

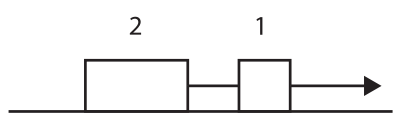
(b) The elevator is accelerating downward at 3 m/s^2.

(c) The elevator is moving upward at a constant velocity of 10 m/s.

(d) The elevator is accelerating upward at 2 m/s^2.

(e) For which scenario does the child feel the heaviest? The lightest?

16. Two boxes sitting on a frictionless surface are connected to one another by a string of negligible mass. Box 1 has a mass of 5 kg and Box 2 has a mass of 10 kg. The boxes are being pulled to the right with a constant force of 3 N.

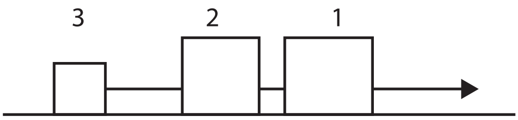


(a) Draw force diagrams for each box and for the system consisting of both boxes.

(b) Determine the tension in the string connecting the boxes.

(c) Find the acceleration of the system.

17.



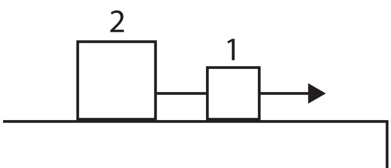
Three boxes sitting on a frictionless surface are connected to one another by strings of negligible mass. Box 1 has a mass of 5 kg and Box 3 has a mass of 2 kg. The boxes are being pulled to the right with a constant force of 22 N and the acceleration of the entire system is 2 m/s^2.

(a) What is the mass of Box 2?

(b) Draw a force diagram for each box and for the system consisting of all three of them.

(c) Determine the tension in each string.

18. Two boxes sitting on a frictionless surface are connected to one another by a string of negligible mass. Box 1 has a mass of 4 kg and Box 2 has a mass of 8 kg. The boxes are being pulled to the right with a constant force of 3 N.



(a) Draw a force diagram for each box and for the system that contains both.

(b) Determine the acceleration of Box 1.

(c) How long does it take Box 1 to reach the end of the table, 2 m away?

19. A rope is causing an object to accelerate upward. Design an experiment to measure the force of the rope. You have a watch, a bathroom scale, and a measuring tape.

20. (a) Can the velocity and the net force ever point in opposite directions? If so, describe a motion where this occurs.

(b) Can acceleration and net force vectors ever point in opposite directions? If so, describe a motion where this occurs.

(c) Is it possible for an object to have zero velocity and also be accelerating? If so, describe a motion where this occurs.

(d) What must be true of the directions of the acceleration and the velocity if an object is speeding up?

(e) What must be true of the directions of the acceleration and the velocity if an object is slowing down?

21. A centrifuge rotates at 40,000 rpm. The bottom of a test tube rotates in it in a circle whose radius is 9 cm. The test chamber contains a sample whose mass is 5 g.

(a) What is the acceleration at the 9 cm radius?

(b) What force must the bottom of the test chamber exert on the sample?

22. A car moves with constant speed in a circular path around a corner on a horizontal road.

(a) What is the force that holds the car in its circular path?

(b) The car's speed is now increased by 50%. By what factor must the horizontal force increase?

23. (a) What is the centripetal acceleration on a 100 kg person at the equator resulting from the earth's rotation?

(b) What percentage of the person's weight is the centripetal force?

(c) What is the interaction that provides the centripetal force?

24. What is the centripetal force on a 100 kg person resulting from the earth's motion around the sun? What percentage of the person's weight is this?

25. A skater has a moment of inertia of 40 kg m^2 when she stretches out her arms. She starts out with an angular velocity of 70 rpm. She then draws her arms in, and her moment of inertia decreases to 10 kg m^2. What is her angular velocity now? (Do you need to convert the units of rpm to SI units?)

26. A beetle whose mass is 2 g sits on a turntable at a distance of 12 cm from the center. The turntable is rotating at 33 rpm.

(a) What are its acceleration and centripetal force?

(b) What is the interaction that provides the centripetal force?

27. A rock at the end of a string is being swung in a circle with higher and higher speed until the

string breaks. Describe the motion after the string breaks.

28. A torque of 30 Nm is applied to an engine shaft that is originally at rest. Its moment of inertia is 8 kg m^2. What is its angular velocity after 10 s?

29. A car engine is rotating with an angular velocity of 100 rpm. It then decelerates with an angular acceleration of 2 rad/s^2. How long does it take to bring the engine to rest?

30. Two spaceships are next to each other, both in the same circular orbit around the earth. The mass of the first is three times the mass of the second. Which of the following are the same and which are different for the two spaceships?

velocity
acceleration
gravitational force
centripetal force

31. A mouse sits on a turntable 20 cm from the center. The coefficient of static friction is 0.2. The angular velocity increases steadily from zero. At what angular velocity does the mouse slide off?

32. Samantha swings a ball at the end of a string in a circle. The ball's mass is 0.2 kg. It is in uniform circular motion with a radius of 1.5 m and an angular velocity of 1.2 rps.

(a) What are the magnitude and direction of the centripetal force?

(b) Draw a vector diagram that shows the centripetal force, the weight, the tension in the string, and the relation between these three vectors.

(c) What is the angle between the string and the horizontal?

(d) How would the angle change if the ball had twice the mass?

33. A geosynchronous orbit is one where a satellite rotates at the same rate as the earth so that it is always over the same spot with respect to the earth. How far is it from the center of the earth? (Start with the force relation. Then find the relation for the period T.)

34. It takes 5 s for the disk of a record player to accelerate from rest to an angular velocity of 33 rpm.

(a) What is the angular acceleration?

(b) How many revolutions does the disk make in this time interval?

35. The wheels on your bicycle have a diameter of 55 cm.

(a) What is their angular velocity in rps when you ride at 15 mph (=6.7 m/s)?

(b) What is the angular acceleration if you reach that speed in 5 s, starting from rest?

(c) How many revolutions does the wheel make in that time?

Multiple choice questions

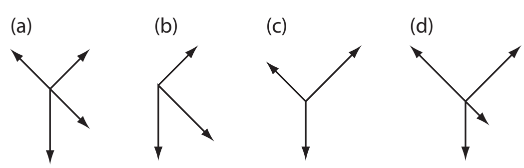
1. While rock climbing, a 50 kg woman falls off a ledge. She is moving down at a speed of 4 m/s when she lands in a bush that stops her fall in 0.40 m. The magnitude of the average force that the bush exerts on her body as she sinks into the bush is closest to

(a) 200 N
(b) 500 N
(c) 2000 N
(d) 1500 N
(e) 1000 N

2. A rope exerts a tension force T on a crate that is initially at rest on a horizontal frictionless surface. When the crate reaches a speed of 6 m/s the tension is abruptly reduced by one-half to $\frac{T}{2}$. Just after the tension is reduced, the speed

(a) decreases abruptly to 3 m/s
(b) continues to increase at half the rate
(c) stops after a short delay
(d) decreases to 3 m/s after a short delay
(e) continues to increase at the same rate.

3. At an amusement park a long slide has a sticky portion at the end designed to slow children down before they reach the end. Which of the force diagrams best represents the children while they are on the sticky portion before they come to a stop?



4. Marsha is pulling the front of a sled with a force of 150 N. The sled is on ice and its mass is 45 kg (that includes the mass of her younger

brother). The rope is at an angle of 20° with the ground. Ignore all effects of friction.

A student represented this situation mathematically as follows, using SI units:
x direction: $150 = 45ax$
y direction: $F_{\text{normal}} - (45)(10) = (45)(0)$

Did the student label anything wrong or forget to include something? Circle all of the corrections that should be made (there may be more than one).

(a) 150 should be 150 sin 20°.
(b) 150 should be 150 cos 20°.
(c) a_y is not zero.
(d) There is a 150 sin 20° term missing in the y direction.
(e) The student did not make any mistakes.

5. Three forces, X, Y, and Z, act on a mass of 4.2 kg. The forces are approximately $X = 2\,\text{N}$ toward the east, $Y = 5\,\text{N}$ acting 45° north of east, and $Z = 3.5\,\text{N}$ acting south.

The direction of the net acceleration is close to being

(a) east
(b) 20° north of east
(c) north
(d) 10° north of east
(e) 44° north of east.

6. When a particle moves in a circle with constant speed, its acceleration is

(a) increasing
(b) constant in direction
(c) zero
(d) constant in magnitude
(e) constant in magnitude and direction.

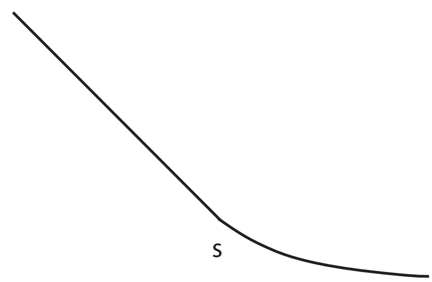
7. A rock is being swung in a circle at the end of a string, with higher and higher speed, until the string breaks. What is the motion of the rock just after that?

(a) spiraling inward
(b) spiraling outward
(c) tangential
(d) radial
(e) none of the above

Synthesis problems and projects

1. What is the mathematical relation that shows that two spaceships in the same circular orbit are moving with the same speed?

2. You are a ski coach estimating the speed of a skier as she approaches a ski jump. You would like to know whether friction is negligible. You know the angle that the slope makes with the horizontal and you know the length, L, of the ramp. You have a digital video of the skier as she goes from rest down the ramp until she makes her jump. You also have a stopwatch and a measuring tape. What can you do to determine whether friction is negligible? What assumptions are you making?



3. A mouse sits on a horizontal turntable that moves with angular velocity ω.

(a) What is the origin of the horizontal force that keeps the mouse from sliding off?

(b) What is the maximum value of that force, i.e., the value beyond which the mouse will slide off?

(c) What is the mouse's acceleration while it is sitting on the turnable?

(d) Write down the relation between the answers to parts (b) and (c) in terms of ω and R, but not v.

(e) The angular velocity at which the mouse begins to slide is proportional to R^n. What is the value of n?