CHAPTER 8

Electricity: It Is Everywhere

We think of the phenomena of mechanics as being everywhere around us, while electricity is more remote and unusual. It turns out instead that it is the electric force that dominates almost everything that impinges on our lives, although in ways that we are not so immediately aware of.

The first thing that comes to mind is the gadgetry of civilization. But more fundamentally, it is the electric forces between atoms that give rise to the existence of molecules, liquids, and solids with their endlessly varied properties, and it is the electric forces within atoms that are responsible for the existence of atoms and for the structure of the elements.

In this chapter we examine the properties of the electric force, and introduce the concept of the electric field, which profoundly changed the way we think about forces.

8.1 The electric force

A world full of charges

We rarely associate everyday experiences with electric forces. We know that electricity causes the shock that we sometimes feel after walking across a carpet, and that it is responsible for lightning. But its sweep goes enormously farther. It is at the root of almost everything that we are aware of, our very existence in all its vast variety.

On the one hand, our civilization depends, in some places almost entirely, on electricity. We use it for light, for heat and cold, and for motors in factories, farms, trains, and households. Without it there would be no communication by telephone, radio, or television, and none of the information technology based on computers.

It is, however, in the microscopic realm that the electric force reigns supreme. If we look at the size scale beyond the nucleus, but below the size of planets and stars, the electrical force is just about the only one that matters. It holds the electrons to the nuclei to form atoms and the atoms to each other to form molecules and solids.

All contact forces, i.e., all the pushing and pulling that we do, is electrical at the point where atom meets atom. All chemical changes and transformations are the result of changes in the position and motion of electrons. This is true as well of all biological processes.

The electrical nature of our civilization is apparent all around us. Each time we switch on the light, or the toaster, or the vacuum cleaner, pick up the telephone or turn on the radio or the television set, we affect the motions of electrons. Each one is so light and small that it is far below any possibility of having an impact on our senses. But they cooperate, and flow through wires to bring about the large-scale, macroscopic effects that we experience.

The electrical nature of matter is far less apparent. All atoms contain protons with their positive charge in the nucleus, surrounded by electrons with their negative charge. Yes, there are also neutrons and neutrinos, but except in the nuclear realm or in the astronomical realm that is so closely related to it, these neutral particles play, at best, a subtle role.

We already saw a drastic example in Chapter 2 that showed how strong the electric force is. We imagined all the protons from just one gram of hydrogen at the north pole, and all the electrons at the south pole, 4000 miles away. The force of attraction between the two turned out to be the enormous force equal to the weight of about 50 tons.

The force between protons and electrons is so large that we rarely encounter a situation where their numbers are not equal. Only under special circumstances can they be separated from one another, at least here on earth, and then only a minute fraction of them.

What we can do is to shift them around with respect to each other. We can move the electrons a little further away from the protons. We can cause them to move differently, so that, on average, the distance between the electrons and protons in a piece of material is just a little different. This is what happens each time a chemical reaction occurs.

The energy that changes when atoms and molecules combine or dissociate is commonly called *chemical energy*. Each such change, whether it is as subtle as in biological processes or as violent as in burning and explosion, is, on the tiny scale of atoms, a change in electric energy, i.e., in the electric potential energy and the kinetic energy of the electrons and atoms.

Electricity and gravitation

The force between two point charges is described by Coulomb's law, $F_e = k\frac{Q_1Q_2}{r^2}$. If we compare it to the force between two masses, as given by Newton's law of gravitation, $F_g = G\frac{M_1M_2}{r^2}$, we see that the two kinds of forces depend on the distance, r, in the same way. In both cases the force between the two interacting objects (charges or masses) varies inversely with the square of the distance between them. That gives us the comfort of familiarity. It also allows us to take over some of the concepts and calculations from the earlier discussion.

But the differences between the two forces are great. Masses always attract, but charges can attract or repel each other. The magnitudes are widely different. The electric force between a proton and an electron is larger by a very large factor (about 10^{40}) than the gravitational force. Hence, where the electric force plays any role at all, its effect is likely to be far more important than that of the gravitational force.

EXAMPLE 1

A proton is attracted to a stationary electron and is accelerated toward it.

(a) What is the acceleration of the proton when they are separated by a distance of 1 m?

(b) As you answered part (a), did you include all of the forces of interaction between the proton and the electron? Why or why not?

Ans.:

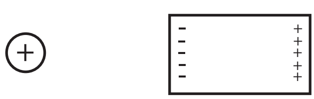
(a) The electric force on the proton is $k\frac{e^2}{r^2}$. The proton's acceleration is $k\frac{e^2}{m_p r^2} = (9 \times 10^9)\frac{(1.6\times10^{-19})^2}{(1.67\times10^{-27})(1^2)} = 0.14\ \text{m/s}^2$.

(b) There is also the gravitational force, $F_g = G\frac{M_p M_e}{r^2}$, and the proton's gravitational acceleration, $a_g = G\frac{M_e}{r^2} = 6.67 \times 10^{-11}\frac{0.91\times10^{-30}}{1^2} = 6.1 \times 10^{-41}$ m/s^2, which is smaller by a factor of about 10^{40} and can be neglected.

Separating charges: polarization

When we talk about mechanical forces on a rock or other object we don't usually think about motion that might result *inside* the object. It is different for electrical forces. Electrons, and to a lesser extent nuclei, can move within atoms and molecules. In metals some of the electrons can move quite freely. The way charges are distributed in objects can therefore change in response to forces on them.

Consider this example: put a positive charge near an uncharged block. The block may not have any *net* charge, but like everything else it is full of charged particles—its electrons and protons. The positive charge outside will repel the positive charges inside, and they will move a little away from it. The negative charges will be attracted and will move closer. The positions of the charges will now be different for the positive charges and for the negative charges. The object has become *polarized*.



Not only that, but the attractive force on the negative charges, which are now closer, is larger than the repulsive force on the positive charges, which have moved away. Hence there is a net attractive force between the uncharged block and the charge outside it!

Charged objects occur in nature as ions, nuclei, and particles such as electrons. Macroscopic objects can be charged by rubbing two dissimilar objects against each other. Electrons can then transfer from one to the other, leaving both objects charged. Some materials, such as hair, lose electrons easily. Others capture electrons easily.

Charges are rarely fixed in space, and actual situations can be quite complicated. Electrons in atoms and solids, for example, are never at rest. As we study the effects that charges have on one another we may have to consider examples and problems that are simplified and not completely realistic.

EXAMPLE 2

Rub a balloon with your hair. Some electrons transfer from the hair to the balloon. Rub a second balloon with a sheet of plastic. Electrons will transfer from the balloon to the plastic.

(a) What happens when the two balloons are brought close to each other?

(b) What happens to your hair?

Ans.:

(a) The first balloon carries an extra negative charge. The second balloon carries an extra positive charge. The balloons attract.

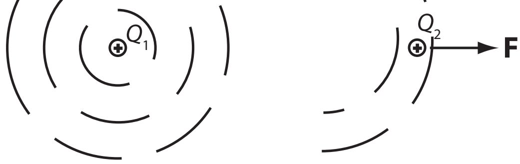
(b) The hair is left positively charged. The individual hairs repel each other and will tend to "stand up."

8.2 The electric field

What is a field?

The idea of the electric field, and of fields more generally, is one of the most powerful that physics has given us. It has changed the way we think of interactions and forces, whether they are electric, gravitational, or of any other kind.

We introduce it in a way that gives barely a hint of its possibilities. One charge exerts a force on a second charge. Let's say the same thing differently: the first charge creates an electric field; the second charge is in that field, and as a result it is acted on by a force.



Nothing is changed *operationally*. Only the force is observable. The field is an artifice, an invention. Yet our perception has changed.

Can we think of another example of an "immaterial" field? Here is one that is more elusive and fanciful: think of a person, first alone. By outlook and appearance, manner and gesture, that person affects his or her environment. If another person enters the "field" of the first one, this second person is affected differently, and as a result behaves differently, depending on the perception of "friendliness," or otherwise, of the first one.

The interaction requires two people, but you can imagine a "friendliness field" created by just one. Similarly, the electric field is there even if you have just one charge. But for a force to exist it takes two.

The first charge, just by its existence, creates a field. The field is everywhere. It is unseen and undetected, until another charge is placed somewhere, and experiences a force as a result of being in the field.

We can use the second charge as a probe, or *test charge*. If it is acted on by a force, it shows that there is a field. We can calculate what the force on the test charge will be once we put it there. After all, we know Coulomb's law. All we need to know is how large the charges are and how far they are from each other.

We define the direction of the field at a point to be the same as the direction of the force there on a positive charge. We define the magnitude of the field as the magnitude of the force divided by the magnitude of the test charge.

EXAMPLE 3

Go to the PhET website (http://phet.colorado.edu) and open the simulation *Charges and Fields*. Select "Run now." Select "grid."

With the mouse, pull a positive charge from its "box" on the right to the middle of the screen. Select "Show E-field." The arrows show the direction of the electric field. They are fainter where the field is smaller. Pull an "E-sensor" from its box to a place near the charge. Its arrow shows the direction and the magnitude of the field. Compare its arrow to the other field arrows. Deselect "show E-field" to see the E-sensor arrow more clearly. Move it around the charge at different distances from the charge to explore the field.

Ans.:
The field is away from the charge. It becomes smaller as you move away from the charge.

In one sense we have done nothing. There isn't anything, so far, that we can describe or calculate by using the concept of the electric field that can't also be done without it. Yet the way we look at what is happening, and the way we think about it, has profoundly changed.

We no longer need to think of the two charges exerting forces on each other across the empty space between them. We now have a way for them to communicate directly. The field is created by a charge and spreads out from it. It is the field that then exerts a force on any other charge in it, right there where the charge is.

The story is similar for the gravitational field. The earth, just by its existence, creates a gravitational field. The field changes the space around the earth. A ball flying through the field is affected by it, and experiences a force. It is no longer a force of the earth, somehow reaching out through empty space. The field provides the communication and interaction between the earth and the ball.

EXAMPLE 4

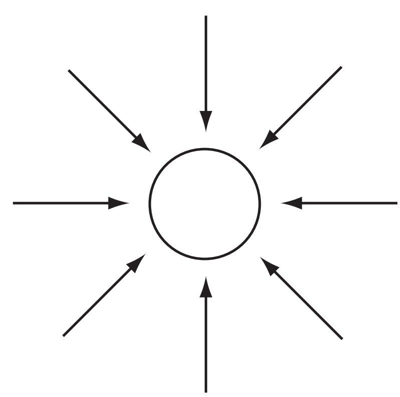
On a sketch of the earth, draw vectors for the force on an object outside it, at eight evenly-spaced places.

(a) Where do they point?

(b) What approximation did you have to make to reach the answer to part (a)?

(c) We define the gravitational field similarly to the way we define the electric field: it is the force on a test mass divided by the test mass. What is the direction of the gravitational field near the surface of the earth?

Ans.:



(a) They point toward the center of the earth.

(b) The force vectors point to the center only if the earth is spherically symmetrical, i.e., if we assume that the earth's density at any point depends only on the distance from the point to the center, and not on the angle, i.e., the density, ρ, is a function only of the radial distance, r: $\rho = f(r)$. In addition, we are assuming that the influence of all other astronomical bodies is negligible.

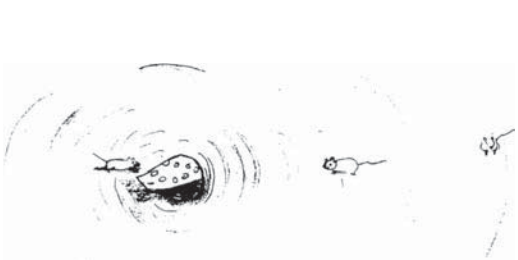
(c) The gravitational force on the test mass and the gravitational field are in the same direction. Since the force is down, toward the center of the earth, the gravitational field is also.

From what we have said so far, we see the electric field as an aid to the calculation of electric forces. That alone would be useful. But there is much more. It turns out that the field has its own independent existence. It is created by a charge, but it can move off, through empty space, in ways that are no longer tied to its origin.

This is the real triumph of the field: electric and magnetic fields can propagate through space. Light waves, radio waves, *electromagnetic waves* of all kinds move through otherwise empty space. They are created by charges, but float off, disembodied, on their own.

The electric field is still what we said it was at the beginning: it tells us that at any point where there is an electric field, there will be an electric force on a charge, if we put one there. Since force is a quantity that has both magnitude and direction, i.e., is a vector quantity, the same is true of the electric field. Quantitatively, if we know the magnitude and direction of the electric field at some point and the sign and magnitude of the charge that we put at that point, we can calculate the force on the charge, exerted by the field.

EXAMPLE 5

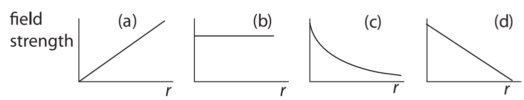


Here is a different kind of field. The cheese creates the field. The mice feel the effect of the field. The field is stronger closer to the cheese, as is shown on the diagram by the greater enthusiasm of the mice as they get closer.

In this case the field consists of molecules released by the cheese and diffusing in the air. This is very different from the electric field, where no atoms or molecules of any material are involved.

(a) Which of the following graphs could describe the field strength as a function of the distance from the smelly cheese?

(b) Which graph could describe the electric field as a function of distance from a point charge, q?



Ans.:
The "cheese field" is very different from an electric field. What it is likely to have in common with the electric field of a point charge is that it decreases as the distance from the "source" increases. The decrease is, however, very unlikely to be linear, with a definite end, as in (d). We reject the increasing and the constant field. This leaves (c), where the field decreases strongly close to the cheese, and then more gradually as the distance from it increases.

The same is true for the electric field of a point charge. In this case we know the way in which the magnitude of the field changes with the distance from the source. We know that the force decreases as $\frac{1}{r^2}$, so that $E \propto \frac{1}{r^2}$.

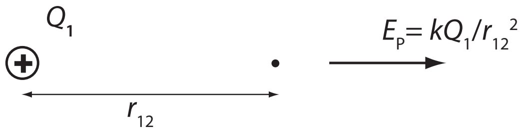
Coulomb's law revisited: force and field

Let's look at Coulomb's law again. We can think of one of the charges, Q_1, as the source of the electric field. The other, Q_2, at the point P, *experiences* the field created by the first charge. It is acted on by the electric force. The force on Q_2 is described by Coulomb's law, and has the magnitude $F_{12} = k\frac{Q_1 Q_2}{r_{12}^2}$, where r_{12} is the distance between Q_1 and Q_2. The direction of the force is such that the two charges repel if they have the same sign and attract if their signs are opposite.

Now separate the force equation into two parts:

$$F_{12} = \left[\frac{kQ_1}{r_{12}^2}\right] [Q_2]$$

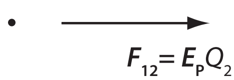
We call the first part on the right-hand side of this relation E_P, the electric field created by Q_1 at the point P, a distance r_{12} from it.



We can now write the force relation as $F_{12} = E_P\, Q_2$.

This is a relation between the *magnitudes* of the force and the field. But both are vector quantities that also have *direction*. We can incorporate the direction by making it a vector equation and letting Q_2 be positive or negative:

$$\mathbf{F}_{12} = \mathbf{E}_P\, Q_2$$



$\mathbf{F}_{12}$ and $\mathbf{E}_P$ are in the same direction if Q_2 is positive and in opposite directions if Q_2 is negative. The direction of the electric field is defined so that a positive charge experiences a force in the same direction as the field and a negative charge experiences a force opposite to the field direction.

EXAMPLE 6

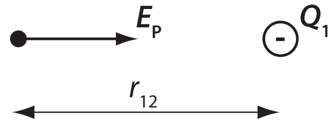
(a) What are the units of E?

(b) What is the electric field of a -5 nC (-5×10^{-9} C) charge at a point 1.2 m to the left of the charge?

Ans.:

(a) The relation $E_P = \frac{F_{12}}{Q_2}$ shows that the units are those of force divided by charge, or N/C.

We can also look at the source relation for E: $E = \frac{kQ_1}{r^2}$. We know from Coulomb's law that the units of k are $\frac{\mathrm{N\,m^2}}{\mathrm{C^2}}$. The units of E are then $(\frac{\mathrm{N\,m^2}}{\mathrm{C^2}})(\mathrm{C/m^2})$ or N/C.



(b) The magnitude is $E_P = \frac{kQ_1}{r_{12}^2} = \frac{(9\times10^9)(5\times10^{-9})}{1.2^2} = 31.25$ N/C. Since Q_1 is negative, the electric field vector is in this case toward the charge, to the right.

EXAMPLE 7

A charge of $+4$ nC is at a point P where the electric field is 20 N/C to the left.

(a) What is the force on the charge?

(b) What do we know about the source of the field?

Ans.:

(a) The charge experiencing the force is 4 nC. Hence the magnitude of the force is $F_{12} = Q_2 E_P = 80 \times 10^{-9}$ N. Since Q_2 is positive, F_{12} and E_P are in the same direction, to the left.

(b) The force on the charge tells us the magnitude and direction of the electric field, but provides no information about the sources of the field.

We have taken the perfectly symmetric Coulomb's law, where each charge plays the same role, and separated it into two parts: the source (Q_1) and the probe (Q_2). The first charge, Q_1, is the source of the electric field, the second charge, the test charge Q_2, probes the effect of the field.

As long as there is no charge anywhere except for Q_1, the field is a figment of our imagination. Only when a second charge (Q_2) is put at some point does the field become observable. There is then a force on Q_2, equal to the field at that point, multiplied by Q_2.

We see that there are two ways to find out what the electric field is. We can either start with the source charge (Q_1) or with the test charge (Q_2).

To use the test charge, we have to know or determine what force, $\mathbf{F}$, it experiences. Since $\mathbf{F} = Q_2\mathbf{E}$, the electric field at a point, $\mathbf{E}$, is equal to the force on the test charge at that point, ($\mathbf{F}$), divided by the amount of the charge Q_2, i.e., $\mathbf{E} = \frac{\mathbf{F}}{Q_2}$. In this case we need to know nothing about the sources of the field.

The second way to find **E** is to start with the sources of the field. If there is only one point-like source charge, Q_1, the magnitude of the field at a distance r_1 from it, at a point P, is $\frac{kQ_1}{r_1^2}$. If Q_1 is positive, the direction of the field is away from the charge, and it is toward Q_1 if the charge is negative.

If there is more than one source charge, each one, Q_n, contributes a field vector at P, a distance r_n away, whose magnitude is $\frac{kQ_n}{r_n^2}$, and whose direction is away from Q_n if Q_n is positive and toward Q_n if the charge is negative. To find the total electric field **E** at P, all of these vectors have to be added up.

EXAMPLE 8

(a) Go to the PhET website (http://phet.colorado.edu) and open the simulation *Charges and Fields*. Select "grid." Put two equal positive charges on a horizontal line 20 divisions apart. Move an E-sensor along that line from the far left to the far right and describe the variation of the field qualitatively. Could you have predicted the direction of the field to the left of both charges, to the right of both charges, and between the charges?

(b) Do the same along the vertical line half-way between the two charges.

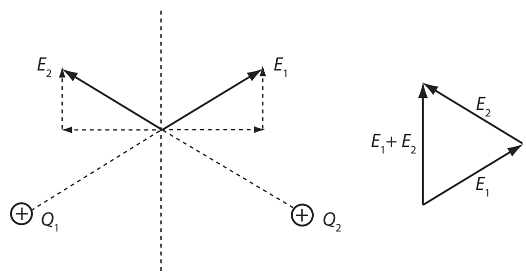
Ans.:

(a) While the sensor is to the left of both charges, the field is to the left. This is as you expect; the force on a positive test charge in this region is the vector sum of two forces, both to the left, one from each of the two charges on the line.

The field becomes very large as you get close to the left charge. It reverses direction and is to the right as you move the sensor to the region between the two charges. As you move the sensor further to the right the field becomes smaller, until it reaches zero in the middle. Again this is what you expect. The force on a test charge is to the right from the left charge and to the left from the right charge. (In both cases it is away from a positive charge.) The two contributions are in opposite directions, and in the middle they cancel. Moving closer to the charge on the right the field becomes larger, as the contribution from the right charge predominates.

In the region to the right of both charges the field is away from both, to the right, and becomes smaller as the sensor is moved further to the right.

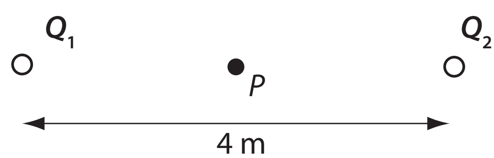
(b) The field is upward above the line and downward below the line.



We already know that on the line, half-way between the charges the field is zero because the contributions to the field from the two charges add up to zero. At other points along the vertical line each contribution to the field has two components: the horizontal components have the same magnitude and are in opposite directions. They cancel. The vertical components are in the same direction, and they add. Alternatively, we can add the vectors representing the fields E_1 and E_2 from the two charges end to end, as on the right side of the figure.

The field is zero half-way between the charges. It also goes to zero far from the charges. Its magnitude therefore has a maximum at some point above the line joining the two charges and at a point symmetrically located below.

EXAMPLE 9



Two equal charges, Q_1 and Q_2, each 8 μC, are 4 m from each other. The point P is midway between the two charges.

(a) Both charges are positive. What is the electric field at P?

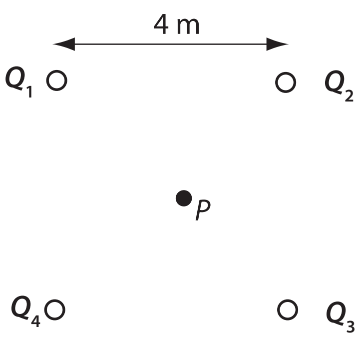
(b) Q_1 is negative and Q_2 is positive. What is the electric field at P now? What is the force on a +5 μC charge at P?

Ans.:

(a) The fields at P of Q_1 (E_1) and Q_2 (E_2) have the same magnitude and are in opposite directions. The two vectors add up to zero, so that the electric field at P is zero.

(b) In this part the fields E_1 and E_2 are in the same direction, to the left. (E_1 is toward Q_1 and E_2 is away from Q_2.) Each has magnitude $\frac{(9\times10^9)(8\times10^{-6})}{2^2} = 1.8 \times 10^4$ N/C, so that the field from both is 3.6×10^4 N/C. The force on a +5-μC charge is $(5 \times 10^{-6})(3.6 \times 10^4)$ or 0.18 N to the left.

EXAMPLE 10



Four equal charges, Q_1, Q_2, Q_3, and Q_4, each 5 μC, are at the corner of a square whose sides are 4 m.

(a) All charges are positive. What is the electric field at the point P in the center of the square?

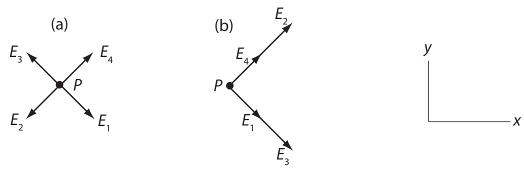
(b) For this part Q_1 and Q_4 are positive and Q_2 and Q_3 are negative. What is the direction of the electric field at P?

(c) What is the magnitude of the field at P in part (b)?

Ans.:

(a) The four vectors E_1, E_2, E_3, and E_4 point away from the four charges. They add up to zero.

(b) This time the two vectors E_1 and E_3 point in the same direction (away from Q_1 and toward Q_3), and so do E_2 and E_4. Their y components



cancel and their x components add, so that the total field at P is in the x direction, to the right.

(c) The distance from any corner to the center of the square is $2\sqrt{2}m$. The field from Q_1 at P, E_1, is $\frac{kQ_1}{(2\sqrt{2})^2} = \frac{(9\times10^9)(5\times10^{-6})}{8} = 5625$ N/C. Its x component is $E_1 \cos 45° = (5625)(0.707) = 3977$ N/C. The x components of E_1, E_2, E_3, and E_4 are the same. The total field at P is therefore $4E_1 \cos 45°$, or 1.59×10^4 N/C.

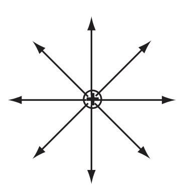
We could use the same procedure to find the field at some other point, not at the center of the square. There would again be four vectors to add, but each would have a different magnitude and point in a different direction. The addition procedure would be much more time consuming. We see that the *symmetry* of the problem that we have solved makes it much simpler.

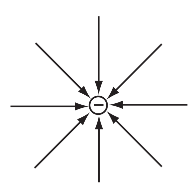
8.3 Field lines and flux

Lines to represent the field

There is a wonderful and productive geometric representation that allows us to get an intuitive but rigorous and detailed feeling for the electric field.

We associate with each positive charge, Q, a number of *electric field lines* emerging from it. The direction of an electric field line, at any point on it, is the same as the direction of the electric field. If the charge is negative, the lines go toward it, so that the lines begin at positive charges and end at negative charges.





So far a drawing of the field lines is a representation that shows the direction of the electric field. We can go further and also incorporate its magnitude. We do this by letting the number of lines divided by the area that they cross (at right angles to the lines) represent the magnitude of the electric field. This means that the lines are closer together where the field is stronger.

Here is why the field concept works: think of an imaginary sphere with a charge at its center. Coulomb's law tells us that the field is proportional to $\frac{1}{r^2}$. The area, A, of the sphere is proportional to r^2. The two factors cancel in the product EA so that the result is independent of r. E is the number of lines through the surface of the sphere, per unit area. EA is therefore just the number of lines through the surface of the sphere. If this number does not depend on r, the number of lines is the same through all possible spheres with this charge at the center, regardless of their radius.

The cancellation of r^2 is the crucial fact that leads to the importance and usefulness of the electric field lines. It is the result of the confluence of two quite separate features, one that is characteristic of the electric field and one that is purely geometric. The first is the power "2" of r in Coulomb's law, so that $E = k\frac{Q}{r^2}$. The second is the power "2" of r in the surface area of a sphere, $A = 4\pi r^2$. It is the fact that these two numbers are precisely the same, so that they cancel, that makes the description of the field by field lines possible. If these two powers were not equal, a set of continuous lines could not describe the field.

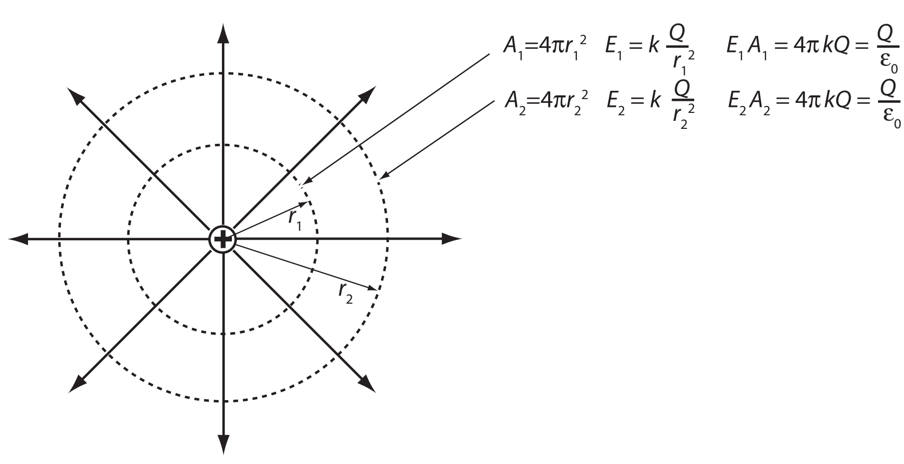
We can look at the same story more precisely: what is the actual number of lines emerging from a single positive charge, Q? Draw an imaginary spherical surface around Q, with radius r. The electric field at the surface of the sphere is perpendicular to the sphere, and its magnitude is $k\frac{Q}{r^2}$, where k is the proportionality constant in Coulomb's law. The surface area, A, of the sphere is $4\pi r^2$. The number of lines through the spherical surface is EA, or $(k\frac{Q}{r^2})(4\pi r^2)$, which is equal to $4\pi kQ$.

k is also often written as $\frac{1}{4\pi\epsilon_0}$, where ϵ_0 is called *the permittivity of free space*. (We will generally just use the symbol, rather than this unwieldy terminology and its numerical value.) The number of lines through the sphere can then be written as $\frac{Q}{\epsilon_0}$.

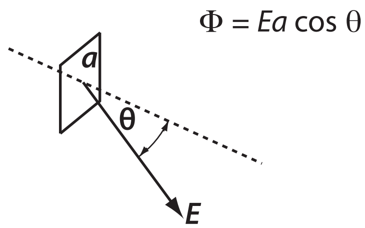
Electric flux

So far we have just one charge, Q, with $\frac{Q}{\epsilon_0}$ lines through the surface of our imaginary sphere, coming from the charge at its center if it is positive and going toward it if it is negative. The number of lines crossing the surface per unit area represents the magnitude of the electric field, and the direction of the lines is the same as the direction of the electric field.

The total number of the electric field lines crossing the surface area of the sphere is EA, where E is the magnitude of the electric field and



A is the surface area of the sphere. The quantity EA is called *the flux of the electric field*, or just the *electric flux*, through the surface of the sphere. We see that the flux of E through the imaginary surface is equal to $\frac{Q}{\epsilon_0}$. The symbol Φ (Greek capital phi) is usually used for it.



For the sphere with the charge at its center the electric field lines are perpendicular to the surface. In cases where they cross at other angles we define the flux as $E_\perp A$, where $E_\perp$ is the component of E perpendicular to the area.

Again, we haven't done anything. We have introduced the terms *electric field* and *flux*, but except for these new words we still have only Coulomb's law, expressed just a little differently. With the introduction of the electric field lines, however, we have a new representation.

EXAMPLE 11

A charge Q is at the center of a cube. What is the flux through the top surface of the cube?

Ans.:
With Q in the center, the sides are located symmetrically with respect to Q. In other words, it doesn't matter which side we call the top surface. The flux through each is the same and is $\frac{1}{6}$ of the total flux of $\frac{Q}{\epsilon_0}$. The flux through any one side is therefore $\frac{Q}{6\epsilon_0}$.

Note that we can answer the question without knowing how large the cube is, or finding E at any point on the surface of the cube. To find the field at every point on one of the square sides, and then to add them up to find the flux, would be a very difficult problem. It is the *symmetry* of the configuration that makes the problem easy. This kind of simplification is characteristic of questions that can be answered by symmetry considerations.

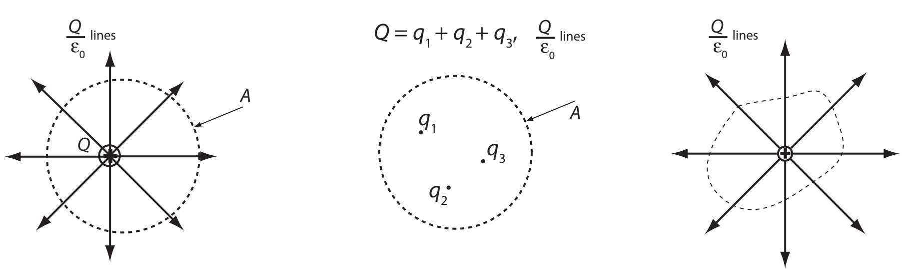
Charge, field, and flux: Gauss's law

The electric field lines are much more than a tool for the visualization of the field. They give us a description that can be vastly generalized. It doesn't even matter whether Q is at the center of the imaginary sphere. The number of lines through the sphere, i.e., the flux of E through it, is still equal to $\frac{Q}{\epsilon_0}$. And if there is more than one charge, this result remains, as long as we let Q represent the total net charge within the sphere. It doesn't matter how many charges there are, where within the sphere they are, or how much of the charge is positive and how much is negative.

In fact, why stick to a sphere? All closed surfaces will have the same property: the flux of E through any closed surface is equal to $\frac{1}{\epsilon_0}$ times the total net charge within the surface. This statement is called *Gauss's law*.

All we have done is to follow Coulomb's law, and let it lead us in a new direction. No additional physical law has entered our development. Gauss's law is entirely equivalent to Coulomb's law. Each can be shown to follow from the other. But Gauss's law gives us an important additional tool for looking at electric fields.

The two laws look very different. Gauss's law allows new calculations and leads to new insights. It, rather than Coulomb's law, is generally considered to be one of the fundamental laws of electromagnetism, known as *Maxwell's equations*.



EXAMPLE 12

A sphere whose radius is R carries a net positive charge Q, uniformly distributed. (This means that for each piece of the sphere with volume ΔV, with a charge ΔQ in it, the *charge density* $\frac{\Delta Q}{\Delta V}$ is the same throughout the sphere. Here the symbol "Δ" denotes a small amount of charge or volume, not, as before, a change in these quantities.)

(a) What is the flux through the surface of the sphere?

(b) What is the electric field at the surface of the sphere?

(c) What is the flux through an imaginary spherical surface outside the charged sphere, a distance r from the center?

(d) What is the electric field at the imaginary surface?

(e) How do the answers to parts (c) and (d) differ from what they would be if all of Q were concentrated at the center?

Ans.:

(a) From Gauss's law the flux through any closed surface is equal to $\frac{Q}{\epsilon_0}$, where Q is the net charge inside the surface. Hence the flux is $\frac{Q}{\epsilon_0}$.

(b) The electric field has the same magnitude at each point on the surface of the sphere and is directed outward. The flux through the surface of the sphere is EA, so that the field is $E = \frac{Q}{\epsilon_0 A}$, where $A = 4\pi R^2$, so that $E = \frac{Q}{4\pi\epsilon_0 R^2}$ or $\frac{kQ}{R^2}$.

(c) $\frac{Q}{\epsilon_0}$.

(d) $\frac{kQ}{r^2}$.

(e) The field lines outside the charged sphere look exactly as they would if all of the charge were at the sphere's center. Therefore the field a distance r from the center (and outside the charged sphere) is the same as if Q were at the center.

The example shows that the number of lines from a single point charge is the same as for a sphere in which this same amount of charge is distributed uniformly throughout the sphere. We can make the result more general: *if the charge is distributed with spherical symmetry (i.e., if the charge distribution looks the same from any angle), the electric field outside the sphere is the same as for the same charge concentrated at the center of the sphere.* The example illustrates how Gauss's law can use symmetry to simplify a problem.

The gravitational field: solving Newton's problem

Use mass instead of charge. Change the proportionality constant. That's all that distinguishes Newton's law of gravitation from Coulomb's law. The essential part is the same: the power of r is still -2. The lines of the gravitational field have all the properties of the lines of the electric field. We know of no negative mass, so that the story is simpler, and the field always points toward a mass. Gauss's law holds for the gravitational field just as it does for the electric field.

We can now use Gauss's law, just as for the electric field, to describe the gravitational field outside a sphere whose mass is uniformly distributed, or more generally, distributed with spherical symmetry. It is the same as if all the mass were at its center.

That the gravitational field outside a spherically symmetric mass distribution is the same as it would be if all the mass were concentrated at the center is the single important result of the application of Gauss's law to gravitation. Of course the earth is not a perfect sphere. Each person walking across the street, each ant, for that matter, destroys that symmetry.

Even the highest mountain in America is smaller than $\frac{1}{1000}$ of the earth's diameter. A greater departure from sphericity is that the diameter from pole to pole is about 43 km less than the diameter of the equator. That's about $\frac{1}{300}$ of the earth's diameter, so the difference is still not great. The density of the earth changes as we go closer to the center, but we ask only for spherical symmetry, not for homogeneity. A homogeneous object has the same properties everywhere. For a spherically symmetric one they may change, but only with the distance from the center. A spherically symmetric object looks the same from any angle. Although the earth (as well as the other planets and moons, and the stars) is not a perfect sphere, the deviations from spherical symmetry are so small that the simple result of Gauss's law for a sphere is sufficient and appropriate most of the time.

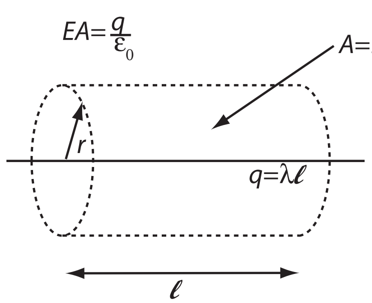
Gauss's law and symmetry

We have seen that if there is some symmetry in a physical situation we can often draw some very general conclusions quite simply. Most of us don't have much experience with this kind of thinking, so it may be helpful to look at some more examples. Problems with symmetric charge distributions are just what Gauss's law is *good* for.

We have just seen the most important one: for a charge Q at a point, or uniformly distributed throughout a sphere or on the surface of a sphere, or, most generally, if the charge density depends only on r, the distance from the center, the electric field outside the charge distribution is directed along the radius, and its magnitude is $\frac{kQ}{r^2}$ or $\frac{Q}{4\pi\epsilon_0 r^2}$. In each of these cases the charge is distributed with spherical symmetry.

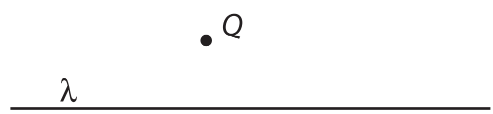
There are two other situations where Gauss's law leads to a simple relation for the field, that of cylindrical symmetry, and that for an infinite plane.

The cylindrical case is that of a long charged line, like a metal rod, or a cylinder, such as a tube or thick cable. The linear charge density is the total charge, Q, divided by the length, L, and is usually called λ (Greek *lambda*), equal to $\frac{Q}{L}$. The electric field is at right angles to the line or to the curved part of the cylinder. (This is exactly true only if the line is infinitely long, but it may be a good approximation for a finite line far enough from its ends.)



Look at an imaginary cylindrical surface of radius r and length ℓ surrounding the line or cylinder concentrically, with an amount of charge Q inside the surface. The surface area of its curved part is $A = 2\pi r\ell$. Because the electric field is at right angles to the curved surface, there is no flux through the flat ends if the cylinder. The total flux through the cylinder is equal to the flux through the curved part of its surface, A, and is equal to EA. From Gauss's law it is equal to $\frac{Q}{\epsilon_0}$, so that $E = \frac{Q}{\epsilon_0 A}$, which is equal to $\frac{Q}{\epsilon_0 2\pi r\ell}$ or $\frac{\lambda}{2\pi\epsilon_0 r}$.

EXAMPLE 13



A wire, 10 m long, is charged uniformly with a charge of 5 μC. A charge of $Q = 3\,\mu\text{C}$ is at a point on a line perpendicular to the wire at its center, 1.5 m from the wire. What is the force on the charge Q?

Ans.:
The question can be answered with the relation that we have derived from Gauss's law if we can assume that the wire is infinitely long. Since the wire is much longer than the distance from the wire to the charge, this is probably a good approximation.

The electric field of the wire, at the point where the 3 μC charge is located, is

$$E = \frac{\lambda}{2\pi\epsilon_0 r}, \text{ or } \frac{2\lambda}{4\pi\epsilon_0 r} = \frac{2k\lambda}{r}$$
$$\lambda = \frac{5\times 10^{-6}\text{ C}}{10\text{ m}} = 0.5\times 10^{-6}\text{ C/m}$$
$$E = \frac{(2)(9\times 10^{9})(0.5\times 10^{-6})}{1.5} = 6\times 10^{3}\text{ N/C}$$

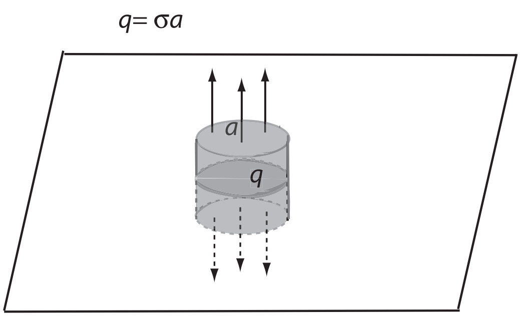
The force on the charge is $F = EQ = (6\times 10^3)(3\times 10^{-6}) = 0.018\text{ N}$.

Because the wire is not infinitely long there are field components parallel to the wire at all points except on the central plane, perpendicular to the wire, on which Q is located. At Q the field is perpendicular to the wire and away from it, and this is also so for the force on Q.

Because the wire is not infinitely long, E will actually be somewhat smaller than the result that we have calculated.

The third case that we will look at is that of an infinitely large plane, charged uniformly with a surface charge density (charge per unit area) of σ. (There are, of course, no infinitely large planes. Our main application will be to the field between two planes, separated by a small distance.) We can tell from the symmetry that E must be at right angles to the surface. Pick a part of the surface, with area a, and surround it on both sides of the plane with an imaginary cylinder. The charge inside the cylinder is then σa.

Because the field is at right angles to the plane there is no flux through the curved part of the cylinder. But lines go out from the plane on both sides of it through the top and the bottom of the cylinder, each with area a, for a total area $2a$. With an electric field magnitude E outward from each surface, the flux out of the cylinder is $(E)(2a)$.



From Gauss's law we know that the flux is equal to $\frac{Q}{\epsilon_0}$, where Q is the amount of charge inside the cylinder, which here is σa, so that the flux is $\frac{\sigma a}{\epsilon_0}$. Putting the two relations for the flux together, we see that $(E)(2a) = \frac{\sigma a}{\epsilon_0}$, or $E = \frac{\sigma}{2\epsilon_0}$.

That's it for Gauss' law. It is always true, but these are among the few situations where it leads to simple results.

EXAMPLE 14

A negative charge of -5 nC is at a point P near a large (assume infinite) single plane that carries a charge with a constant surface charge density of $+2\,\mathrm{nC/m^2}$.

(a) Is it possible to find the electric field at a point near the plane without knowing how far the point is from the plane?

(b) What are the magnitude and direction of the field at P?

(c) What is the force on the charge?

Ans.:

(a) The electric field near an infinite plane is constant and equal to $\frac{\sigma}{2\epsilon_0}$, which is independent of the distance from the plane.

The field lines are perpendicular to the plane. Their density (the number crossing a unit area), which is equal to E, does not change.

Qualitatively, the further away from the plane a point is, the greater the contributions to the field from distant points on the plane. Since the plane is assumed to be infinitely large, there will be distant regions that will contribute, no matter how far away from the plane a point is.

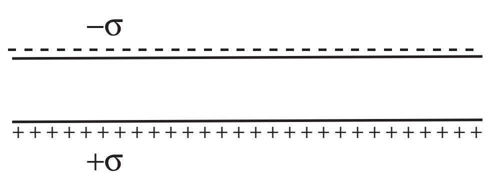
(b) $E = \frac{\sigma}{2\epsilon_0}$, with lines away from the plane and perpendicular to it. $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$, so that $\frac{1}{\epsilon_0} = 36\pi \times 10^9$ SI units.

$$E = (2 \times 10^{-9})(\frac{1}{2})(36\pi \times 10^9)$$
$$= 36\pi \text{ or } 113\,\mathrm{N/C}$$

(c) $F = EQ = (113)(5 \times 10^{-9}) = 5.5 \times 10^{-7}$ N.

The direction of the force is opposite to the direction of the field because the charge is negative. It is toward the plane.

EXAMPLE 15



Two large metal planes are separated by a distance of 2 cm. The upper one is charged negatively and the lower one positively, each with a surface charge density of $5\,\mathrm{nC/m^2}$.

(a) Describe the electric field lines.

(b) What is the electric field between the planes?

(c) What is the electric field away from both planes?

Ans.:

(a) Electric field lines start at positive charges and end at negative charges. (Although we have talked about isolated charges, for each positive charge there is an equivalent negative charge somewhere.) In this case the lines go from the positively charged plane to the negatively charged plane.

Alternatively, we can add the field contributions from the two planes. Between them there is a field $\frac{\sigma}{2\epsilon_0}$ upward from the positive plane and a field equally large toward the negative plane. Together the two fields add up to a field $\frac{\sigma}{\epsilon_0}$ upward.

(b) $E = \frac{\sigma}{\epsilon_0} = (5 \times 10^{-9})(36\pi \times 10^9) = 180\pi = 5.65 \times 10^2$ N/C.

(c) In the region above both planes there is a field down toward the negative plane and a field up away from the positive plane. They have the same magnitude, and add up to zero. Similarly, the field is zero below both planes.

The configuration described in this example is called a *capacitor*. It stores charge on each of the two plates. It also stores the uniform electric field in the region between the plates. Since the problem states that the plates are "large," we have tacitly assumed that we can use the approximation that they are infinitely large. This is a good approximation as long as the distance between the plates is small compared to their length and width. The approximation is best in the central region and least appropriate near the edges, where the field lines curve outward. The field near the edges, where it is no longer uniform, is called the *fringing* field.

8.4 Summary

The electric force is the second of the fundamental forces of nature. While Newton's law describes the interaction between bodies that have mass, Coulomb's law describes the interaction between charges. The gravitational force is always attractive, but the electric force can be one of attraction or repulsion. The electric force depends on the magnitude and the sign of the charges and on how far apart they are from each other: $F_e = k\frac{Q_1 Q_2}{r^2}$.

The attractive force between the negatively charged electrons in the atoms and the positively charged protons in the nucleus of the atom holds the atom together and is responsible for the structure of all atoms, molecules, and their combinations.

An object is *polarized* when the average positions of the positive and negative charges in it are not the same.

We can describe the electric force by using the concept of the *electric field:* each charge creates an electric field that surrounds it. Another charge that is in the field experiences a force. The electric field is a *vector* quantity. Its calculation can quickly become complicated as the number of charges grows. We therefore looked at some of the simplest charge distributions, particularly those where we can make use of *symmetry* to simplify the calculations.

A charge Q_1 has an electric field around it with magnitude $E_1 = k\frac{Q_1}{r^2}$. The direction of the field is away from a positive charge and toward a negative charge.

If there are several charges, each contributes to the field. The total field at a point is equal to the vector sum of the field vectors from each charge.

A mass m has a gravitational field around it, of magnitude g, toward the mass.

In an electric field E_1 a charge Q_2 experiences a force $E_1 Q_2$. The force is in the direction of the field if Q_2 is positive and in the direction opposite to the field if the charge is negative.

As an aid to visualizing the field we use *electric field lines*. The lines begin at positive charges and end at negative charges. The direction of the lines is the direction of the electric field.

When a field, E, is perpendicular to an area, a, the electric flux through the area is Ea. If the normal to the area (the line perpendicular to a) makes an angle θ with the field, the flux through the area is $Ea \cos\theta$.

The flux (Ea) from a point charge, or the number of field lines coming from a point charge, is $\frac{Q}{4\pi\epsilon_0 r^2}4\pi r^2$, or $\frac{Q}{\epsilon_0}$. Here $k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$. This is also the flux (or the number of lines) emerging from a closed surface that contains a net charge Q. (This statement is called *Gauss's law*.)

The field outside a spherically symmetric charge distribution is the same as that of a point charge (with the same total charge) at its center.

The field on each side of a positively charged, infinitely large plane is $\frac{\sigma}{2\epsilon_0}$. If the field is only on one side, its magnitude is $\frac{\sigma}{\epsilon_0}$. This is also the field inside a (infinitely large) capacitor.

8.5 Review activities and problems

Guided review

1. A proton is released from rest at a distance of 0.5 m from an electron.

(a) What is its initial acceleration?

(b) What is the ratio of this acceleration to the gravitational acceleration that results from the gravitational force between the two particles?

2. Go to the PhET website and open the simulation *Balloons and Static Electricity*. The following should be checked as you start: "show all charges," "ignore initial balloon charge," and "wall."

(a) Drag the balloon to the sweater and rub it against it. Move it back to where it started and let go. What happens? Why?

(b) Move the charged balloon toward the wall. What happens to the charges in the wall? What happens when you let go of the balloon near the wall? Why?

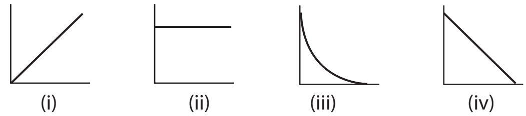
(c) Click on "two balloons." Charge both of them. Get one of them to move slowly in the middle. Can you push it toward the wall with the other balloon? What is meant by *polarization*? Describe the polarization of the wall.

3. Go to the PhET website and open the simulation *Charges and Fields*. Pull out a charge. Use the E-field numbers and the tape measure to find the magnitude of the charge. (The unit V/m is the same as the unit N/C.)

4. (a) On the sketch of a positively–charged sphere, draw vectors for the force on another positive charge outside the sphere at eight places, each at the same distance from the sphere.

(b) Repeat for a negative charge in the vicinity of the same positively–charged sphere.

5. (a) Which of the graphs best describes the magnitude of the gravitational field as a function of distance, going from the surface of the earth to a point half-way to the moon?



(b) Which one best describes the magnitude of the gravitational field as you go from the earth's surface to a height of 20 m?

6. (a) What are the magnitude and direction of the electric field 0.6 m to the right of a +5 nC charge?

(b) Compare the magnitude and direction of the field to the result of Example 6, and explain the difference.

7.

There is a negative charge at the point A and a positive charge at the point B.

A charge of -3 nC is at the point P, where the electric field is 15 N/C to the left.

(a) What are the magnitude and direction of the force on the charge at P°?

(b) Which of the following statements is correct?

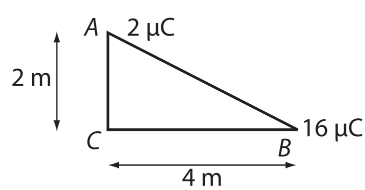
(i) The net charge at A is greater than the one at B.

(ii) The net charge at B is greater than the one at A.

(iii) It is not possible to conclude anything about the relative magnitudes of the charges.

8. Two charges are 3 m apart. $Q_1 = 1\,\mu$C is on the left and $Q_2 = 4\,\mu$C is on the right. Where is the point P at which the electric field is zero?

9.



The figure shows a right-angled triangle with positive charges of 2 μC and 16 μC at the corners A and B.

(a) What is the electric field at C?

(b) What is the force on a 10 μC charge at C?

10. Four charges, each of $5\,\mu C$, are at the corners of a square with 4 m sides, as in the example.

(a) All charges are positive. What are the magnitude and direction of the electric field halfway between Q_2 and Q_3?

(b) The configuration is changed so that the magnitudes of all charges remain the same, but Q_1 and Q_4 are negative and Q_2 and Q_3 are positive. How does the field halfway between Q_3 and Q_4 compare with that found in part (a)?

11. A charge Q is at the center of a regular dodecahedron. (This is a polygon with 12 equal faces, each of which is a regular pentagon.) What is the flux through one of the faces?

12. A sphere with radius R carries a net positive charge, Q, uniformly spread over its surface.

(a) What is the electric field at the sphere's surface?

(b) What is the flux through an imaginary spherical surface just outside the sphere?

(c) What is the flux through a spherical surface surrounding the sphere at $r = 2R$?

(d) What is the electric field at this spherical surface?

(e) How are the answers to parts *a* to *d* different if the same charge, Q, is spread uniformly throughout the sphere's volume?

13. A charge of $Q = 3\,\mu C$ is 2 m from an 18 m-long wire. It experiences an electric force of 0.012 N. What is the total charge on the wire?

14. A charge of $+4\,nC$ and mass 2×10^{-15} kg is held at rest at a point P near a large (assume infinite) plane with a constant charge density of $\frac{3\,nC}{m^2}$.

(a) If the charge is let go, in which direction will it move?

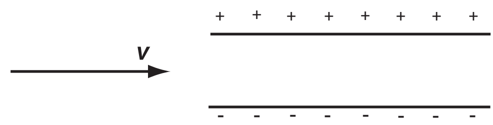
(b) What is its acceleration?

15. Each of the two parallel plates of a capacitor has a surface charge density whose magnitude is $\frac{8\,nC}{m^2}$.

(a) Draw a sketch of the plates to show the charge distribution and the electric field lines.

(b) What is the electric field between the plates?

(c) Describe the path of an electron that is shot into the capacitor along a path parallel to the plates half-way between them.



Problems and reasoning skill building

1. When you run a plastic comb through your dry hair it becomes electrically charged as electrons move to it from your hair. Small pieces of paper are then attracted to the comb. Describe the sequence of movements of charge that leads to this attraction.

2. (a) How many electrons are there in a (neutral) molecule of water?

(b) How many electrons are there in a liter ($= 1000\,cm^3 = 10^{-3}\,m^3$) of water? (The density of water is $1\,g/cm^3$. The molar mass is 18 g, i.e., the mass of N_A (Avogadro's number) molecules is 18 g.)

(c) What is the charge (in C) of all the electrons in a liter of water?

3. A sphere has a charge of $+4\,\mu C$. An additional 6×10^{13} electrons are placed on the sphere. What is the net charge now?

4. A charge $Q_1 = 6\,\mu C$ is at the origin of an $x-y$ coordinate system. A charge $Q_2 = 4\,\mu C$ is at the point ($x = 1.5$ m, $y = 0.6$ m) and a charge $Q_3 = -4\,\mu C$ is at the point ($x = 1.5$ m, $y = -0.6$ m).

(a) What are the magnitude and direction of the force of Q_2 on Q_1?

(b) What are the magnitude and direction of the force of Q_3 on Q_1?

(c) Draw a vector diagram of the forces in parts (a) and (b) and their sum.

(d) What are the magnitude and direction of the net force of both Q_2 and Q_3 on Q_1?

5. A charge is located on the x-axis at $x = 0$. At $x = 1$ m the magnitude of the electric field, **E**, is 1 N/C. Make a graph of E as a function of x from $x = 0.4$ to $x = 5$.

6. (a) Use Coulomb's law to determine the units of k.

(b) What is the magnitude of the electric field at a distance of 10^{-9} m from a proton?

7. A charge of $-3\,nC$ experiences an electric force of 10^{-7} N to the right. What are the magnitude

and direction of the electric field at the point at which this charge is located?

8. At every point on the surface of a sphere of radius 0.4 m the electric field is radially outward, with a magnitude of 20 N/C. What is the flux through the spherical surface?

9. The flux through an imaginary spherical surface is 12 Nm^2/C.

(a) What can you conclude about the charge within this surface?

(b) What can you conclude about the charge within the spherical surface if, in addition, the electric field is radial, with the same magnitude at every point on the surface?

10. (a) In Example 10, part (a) there are four $+5\,\mu C$ charges at the corners of a square, and the field is zero at the center. What changes can you make in the number, the magnitudes, and the positions of the four charges that will leave this result ($E = 0$) unchanged? What is the most general change that you can think of?

(b) For the charges as in part (b) of Example 10, what are the magnitude and direction of the field midway between Q_2 and Q_3?

11. There is a uniform electric field on both sides of a large plane. Its magnitude is 80 N/C and its direction is toward the plane, on each side of it. What is the surface charge density on the plane?

12. A 100 kg sphere and a 100 g sphere outside it are held fixed with the distance between their centers equal to 0.5 m. Each carries a charge of $+2.5\,\mu C$. Assume that the charge on each sphere is and remains uniformly distributed.

(a) What are the magnitude and direction of the force on each of the spheres?

(b) What is the acceleration of each when they are released?

(c) What will you observe when they are released?

13. Two equal positive charges, Q_1 and Q_2, each $5\,\mu C$, are 4 m from each other. The point P is between the two charges on the line joining them, 1 m from Q_1. What are the magnitude and direction of the field at P?

14. There is a uniform electric field of 2×10^4 N/C in the x direction. A point charge of $4\,\mu C$ is placed at the origin. What are the coordinates of the point or points where the total electric field from both the point charge and the uniform field is zero?

15. Three equal charges, each $+4\,\mu C$, are at three corners of a square whose sides are 2 m long. Add up the vectors representing the contributions from the three charges to find the magnitude and direction of the field at the fourth corner of the square.

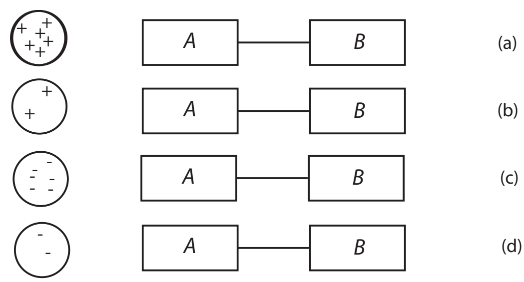
16. The two opposite surfaces of a cell membrane act like the plates of an empty parallel plate capacitor. Assume that the charge densities on the two surfaces are $\pm 6.5 \times 10^{-6}$ C/m^2.

(a) What do you need to assume before you can calculate the electric field within the membrane?

(b) With these assumptions, what are the magnitude and direction of the electric field within the membrane?

(c) What are the magnitude and direction of the force on a singly-charged ($q = +e$) ion within the membrane?

17.



Two metal blocks, A and B, are attached to each other with a removable metal wire. A charged object is brought near block A. The wire is then removed. Rank the four situations shown in the figure in order, with the largest positive charge remaining on B ranked first, to the most negative last.

Multiple choice questions

1. If the distance between two positive ions is doubled, and the charge on each is also doubled, what will happen to the force on each?

It will

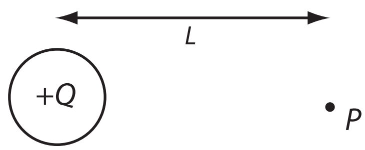
(a) stay the same,

(b) increase by a factor of 2,

(c) decrease by a factor of 2,

(d) decrease by a a factor of 4.

2. The point P is a distance L from a uniformly-charged sphere. The electric field at P is E. What is the value of the electric field for each of the following changes.



(i) Q is doubled.
(ii) L is doubled.
(iii) Both Q and L are doubled.
(a) $2E$
(b) $\frac{1}{2}E$
(c) $4E$
(d) $\frac{1}{4}E$

3. An electron is brought to a point 1 cm from a positively-charged Na^+ ion. A proton is brought to a point 2 cm from an SO_4^{2-} ion.

(a) The force on the electron is half as large as that on the proton.

(b) The force on the electron is four times as large as that on the proton.

(c) The force on the proton is half as large as that on the electron.

(d) The force on the proton is four times as large as that on the electron.

4. Which of the following best describes why the sublimation (transformation to the gaseous phase) of "dry ice" (solid CO_2) is not increased in a microwave oven.

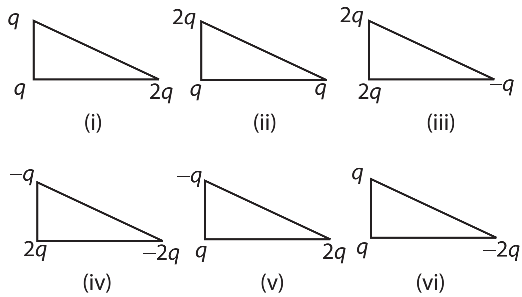
(a) Dry ice is too cold for microwaving.

(b) There are no water molecules in dry ice.

(c) Microwaves do not work with frozen substances.

(d) Adding thermal energy does not affect the rate of sublimation.

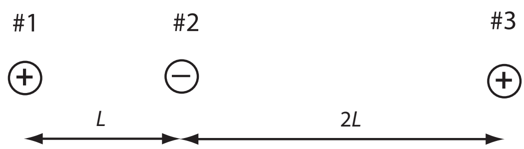
5.



The triangles in the figure all have the same size, 1 unit for the vertical side and 2 units for the horizontal side. The charges are shown on the figure.

Rank the triangles in order of the magnitudes of the net force on the charge in the lower left corner of each triangle.

6.



The figure shows three fixed charges, two of them positive and one negative. The net force on #3 is zero. Which of the following must be true? (There may be more than one.)

(a) #1 carries more net charge than #3.
(b) #3 carries more net charge than #2.
(c) #1 carries more net charge than #2.
(d) #2 carries more net charge than #1.
(e) #3 carries more net charge than #1.