More on Electricity: From Force to Energy, from Field to Potential

To describe the forces between charges at rest there is only Coulomb's law (or its equivalent, Gauss's law). But by using the concept of energy instead of that of force we gain two great advantages, just as we do when we go from gravitational force to gravitational potential energy. One is a mathematical advantage, the other is a physical one.

The mathematical advantage is that force is a vector quantity and energy is a scalar. Adding scalars is easy. It's just like adding numbers. Adding vectors is more involved and takes more effort. The physical advantage depends on a physical law, the law of conservation of energy.

If we use energy we don't need to know what the forces are at every point and at every moment. We can look at just two points along the path of a particle, say A and B. If no energy has been given to the particle between these two points, or taken from it, then its energy at A must equal its energy at B.

More generally, if we use the concept of energy, it doesn't matter what happens to a system between start and end. It doesn't matter what the paths of its constituents are. All we need to know is how much energy, if any, has been added to the system or taken away from it. Moreover, the law of conservation of energy is not limited to physics. It is a great unifying principle that plays an important role in every part of science.

9.1 Electric potential energy and electric potential

The electric potential

We have seen that the concept of the electric field takes us beyond the knowledge of the force between two charges. It allows us to describe the effect on a charge by many charges. It tells us how the presence of one or more charges affects the space around them. If we know the magnitude and direction of the electric field at a point, it allows us to *predict* the force on any charge that we might put there.

We would now like to make use of the advantages that we get if we use energy rather than force. Let's remind ourselves of the definition of potential energy: the work done on a system against the gravitational force is equal to the increase in its gravitational potential energy, and the work done against the electrical force is equal to the increase of its electrical potential energy. (The definition tells us only about changes in energy. Just as for gravitational potential energy, if we want a definite, *absolute* value, we need to choose a reference level where the potential energy is zero.)

We can define a new construct, one that is related to the electric potential energy in the same way that the electric field is related to the electric force. It is the *electric potential* and it is equal to the electric potential energy that a charge has at a point, divided by the value of the charge.

That gives us two quantities that describe the space around a charge: the electric field and the electric potential. They provide information about two properties of the charge, namely the electric force on it and its electric potential energy. One is a vector quantity, the other is a scalar quantity. Together they give a more complete description than either one alone.

Our new quantity, the electric potential, is a property at a point in space. It is defined as the electric potential energy that a charge Q has (or would have) at that point, divided by the amount of that charge, Q. If the potential energy of a charge of Q coulombs is P joules, then the electric potential, V, is $\frac{P}{Q}\,\frac{\text{joules}}{\text{coulomb}}$. If Q is positive, P and V have the same sign, but if Q is negative, they will have opposite signs.

The unit J/C is called a *volt* (V). A difference in electric potential is also called a *voltage difference*. When we buy a nine–volt battery, for instance, it means that there is a potential difference, or a voltage difference, of nine volts between the two terminals of the battery. (This is the "nominal" value, i.e., it is approximate, and decreases with time as the battery becomes exhausted and the chemical processes within it cease.)

A volt is a joule per coulomb. For each coulomb of charge that moves through the battery, a joule of energy is transferred from the battery (where it was stored as internal energy) to the electrons as electric potential energy.

One more reminder: we often talk about the gravitational potential energy of an *object* in the vicinity of the earth, even though we know that it is really the potential energy of the *system* containing the object and the earth. Similarly we say that a charged object has electric potential energy. Again it is the system that we are talking about, containing the charge on which we are focusing, and perhaps other charges that have an effect on it.

EXAMPLE 1

A rock whose mass is 0.5 kg is lifted through a distance of 3 m.

(a) What is the force on the rock exerted by the gravitational field?

(b) What is the magnitude of the work done against the field when it is lifted?

(c) What is the increase of the gravitational potential energy when the rock is lifted?

A charge of 0.5 nC is in a uniform electric field of 12 N/C. It is moved a distance of 3 m in the direction opposite to the field direction.

(d) What is the force on the charge by the electric field?

(e) What is the magnitude of the work done against the field as the charge is moved?

(f) What is the increase in the electric potential energy when the charge is moved?

(g) What is the increase in the electric potential between the beginning point and the endpoint of this path?

Ans.:

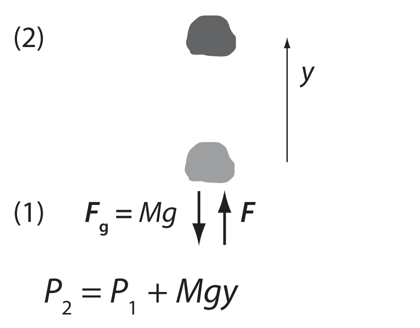
(a) $F_g = Mg = (0.5)(9.8) = 4.9\,\text{N}$.

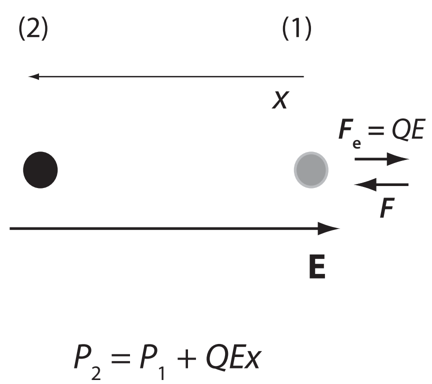
(b) The force, F, moving the rock is in the direction opposite to the force of the gravitational field, but they have the same magnitude, 4.9 N. The work done is $Fh = (4.9)(3) = 14\,\text{J}$.

(c) From the definition it is the same as the answer to part (b), namely 14.7 J.

(d) $F_e = QE = (0.5 \times 10^{-9})(12) = 6 \times 10^{-9}\,\text{N}$.

(e) The force moving the charge is in the direction opposite to the force of the electric field, but they have the same magnitude, 6×10^{-9} N. The work done against the electric field is $(6 \times 10^{-9})(3) = 18 \times 10^{-9}\,\text{J}$.





(f) From the definition of electric potential energy the answers to parts (e) and (f) are the same, namely, 1.8×10^{-8} J.

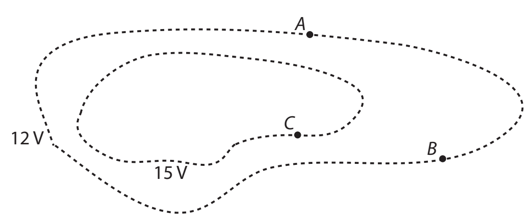
(g) The difference in electric potential is equal to the difference in the electric potential energy of a charge divided by the charge, or $\frac{18\times10^{-9}}{0.5\times10^{-9}} = 36\,\text{V}$.

Equipotentials

If we want to show how high points on a map are, we draw *contour lines*. These are lines that connect points that are at the same height, i.e., points that have the same gravitational potential energy. Similarly, on a map of charges, we can draw *equipotentials*, lines that connect points that are at the same electric potential.

What happens if we move a charge along an equipotential? The potential energy remains the same, and no work is done against the electric force. That must mean that there is no component of the electric field along the equipotential. Since the field has no component along the equipotential, it must be perpendicular to it. Along the field lines, perpendicular to the equipotentials, the potential energy and the potential change more quickly than along any other direction.

EXAMPLE 2



The figure shows two equipotentials, one at 12 V and the other at 15 V.

What is the work that needs to be done to move a 9 nC charge from A to B? From A to C?

Ans.:

A and B are on the same equipotential. They are at the same electric potential and it therefore takes no work against the electric force to move them from A to B.

C is at a higher potential, and it therefore takes work to move the charge there: it is $Q\Delta V = (9\,\text{nC})(3\,\text{V}) = 27 \times 10^{-9}\,\text{J}$.

EXAMPLE 3

Go to the PhET website (http://phet.colorado.edu) and open the simulation *Charges and Fields*. Select "grid" and "show numbers."

(a) Pull one positive charge and one negative charge with the mouse to points along a horizontal 3 m apart, equidistant from each side. Select "show E-field." Drag the blue voltage sensor so that its crosshairs are at points 0.5 m apart along a vertical line first through one of the charges and then through the other. At each point (except right at the charges) click "plot" to draw the equipotential that includes this point. Explore other points. If you select "direction only" it is easier to see the field vectors, but the magnitude is no longer indicated by the intensity of the arrow color.

Turn off "Show E-field" and pull out an "E-field sensor." Drag it around the screen and observe the field arrow. What is its orientation with respect to the equipotentials?

(b) Clear all and repeat, this time with both charges positive and 1 m apart. Describe the equipotentials close to one charge, and then far from both.

(c) Explore other charge configurations.

Ans.:

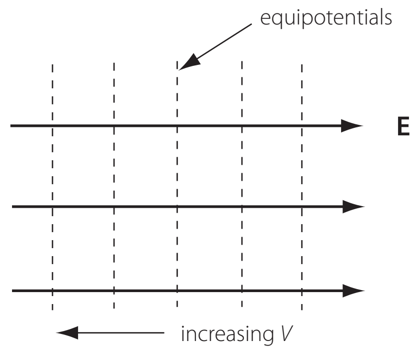
(a) The electric field is a vector field. The red arrows on the screen show the electric field. Both the magnitude and the direction of the field can be seen with the "E-field sensor." The field is strongest close to a charge, pointing away from a positive charge and toward a negative charge. The equipotential lines are perpendicular to the field lines.

(b) Close to one charge the equipotentials surround it. Far from both the equipotentials surround both charges.

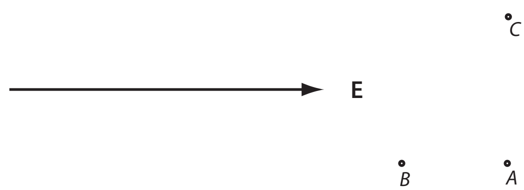
Simple and ideal: the uniform field

An electric field, **E**, is said to be *uniform* in a region where it has the same magnitude and direction at every point. No matter where in this region we put a charge of Q coulombs, the electric force on it is the same: $\mathbf{F} = Q\mathbf{E}$ newtons, with the force in the same direction as the field if Q is positive and in the opposite direction if Q is negative. A good example of a nearly uniform field is that between the plates of a capacitor, as in the last example of Chapter 8.

If we move a positive charge along a field line in the direction opposite to the field, we have to do work against the electric force (against the field) and both the electric potential energy of the charge and the electric potential increase.



EXAMPLE 4



There is a uniform field of 5 N/C in a region. A charge of 2×10^{-4} C is moved from a point A to a point B along an electric field line in the direction opposite to the field, a distance of 3 m.

(a) What is the electric force on the charge at A?

(b) What is the electric force on the charge at B?

(c) What is the work done on the charge, against the electric field, as it is moved from A to B?

(d) Which is larger, the electric potential energy at A (P_A) or at B (P_B)?

(e) What is the difference in the potential energies, $P_B - P_A$?

(f) What is the potential difference, $V_B - V_A$?

The point C is 4 m from A, in a direction at right angles to the field. Let the potential at A be zero.

(g) What is the potential at B and at C?

(h) What is the work that is done on a 2×10^{-4} C charge to move it from C to A?

(i) What is the work done on this charge to move it from C to B?

Ans.:

(a) $F_e = QE = (2 \times 10^{-4}\,\text{C})(5\,\text{N/C}) = 10^{-3}\,\text{N}$. The direction of the force on this positive charge is in the same direction as the field, **E**.

(b) Because the field is uniform, $E_A = E_B$, and the forces are the same.

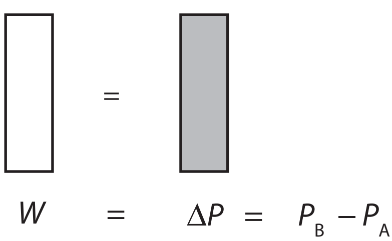
(c) The work done is $F_e \Delta x = 10^{-3}\,\text{N} \times 3\,\text{m} = 3 \times 10^{-3}\,\text{J}$ or 3 mJ.

(d) As work is done against the field, the potential energy increases. It is therefore larger at B than at A.

(e) The difference in potential energy between points A and B is equal to the work done against the field as the charge is moved from A to B. It is equal to 3 mJ.

(f) $V_B = \frac{P_B}{Q}$, $\quad V_A = \frac{P_A}{Q}$. $\quad V_B - V_A = \frac{P_B - P_A}{Q} = \frac{3 \times 10^{-3}\,\text{J}}{2 \times 10^{-4}\,\text{C}} = 15\,\text{V}$.

(g) The line from A to C is perpendicular to the electric field. Therefore there is no change in potential between A and C. The points A and C are on an equipotential.



Now find the potential at B. The work done on a charge is equal to the force on the charge times the distance that the charge is moved. On a unit charge (a charge of 1 C) the force is equal to the electric field, E, and the work done on it is the increase in the electric potential. Here $V_B - V_A = E\Delta x$.

The potential at B is higher than at A by $\Delta V = E\Delta x = (5\,\text{N/C})(3m) = 15\,\text{J/C}$ or 15 V. To determine the actual value of the potentials at B and C we have to decide on a reference level. Here $V = 0$ at A, so that $V_C = 0$ and $V_B = 15\,\text{V}$.

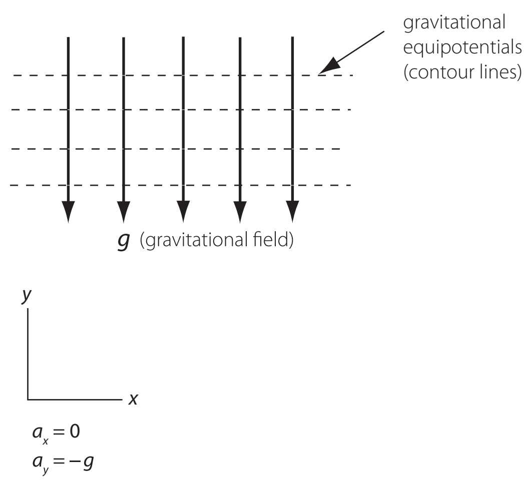
(h) It takes no work to move a charge along an equipotential.

(i) Since the work to move the charge from C to A is zero, the work to move it from C to B is the same as to move it from A to B, i.e. $(15\,\text{J/C})(2 \times 10^{-4}\,\text{C}) = 3 \times 10^{-3}\,\text{J}$. This is so regardless of the particular path from C to B.

(We have assumed that the force of the electric field is the only one that has to be overcome as the charge is moved. If there are other forces, such as gravitation or friction, they need to be taken into account. The values of the potential and the potential differences will still be the same.)

The uniform gravitational field

Near the earth the gravitational force on an object of mass M is downward and equal to Mg. The gravitational field is the force divided by the mass, also downward, and it is equal to g. We can make the approximation that the field is uniform if we ignore the curvature of the earth's surface. In other words, we are confining ourselves to distances so small compared to the radius of the earth that we can treat the earth as though it were flat. Within this approximation the gravitational field can then be considered to be uniform, with lines of the gravitational field that are parallel, equally spaced, and downward.



When we move an object upward, we have to do work against the gravitational force. Both the gravitational potential energy of the object and the gravitational potential increase. Since we know of no negative masses the complication of having two possible signs (as for positive and negative charges) does not arise in the gravitational case.

Motion in a uniform field

We know what happens when we release an object from rest in a gravitational field: it experiences a force (F_g) in the direction of the field (downward), whose magnitude is Mg. Its acceleration is also in the direction of the field, and its magnitude is g. We can write down the expressions for the velocity (v) and the displacement (y). If we use the field direction (down) as positive, they are $v = gt$ and $y = \frac{1}{2}gt^2$.

We can do the same to describe the motion of a charge in a uniform electric field: the force (F_e) is equal to QE. This time it is in the direction of the field only if Q is positive, and it is in the direction opposite to the field if the charge in negative. The acceleration is in the same direction as the force and has the magnitude $\frac{QE}{M}$. (In the gravitational case there is a term M both in the numerator and the denominator, and they cancel.) The velocity is $v = at$ and the displacement is $\frac{1}{2}at^2$.

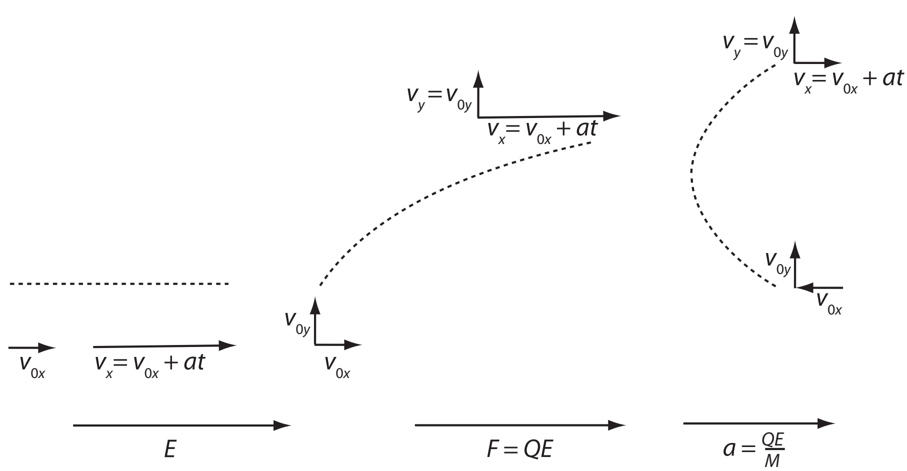
What if the start is not from rest but with an initial velocity v_0? The force and the acceleration remain the same. But just as in the gravitational case the nature of the motion of a charge in an electric field is different depending on the direction of v_0. If v_0 is along a field line it's easy: $v = v_0 + at, \quad x = v_0 + \frac{1}{2}at^2$, and the motion is along a field line. As usual, we have to choose a direction to be positive and keep track of the plus and minus signs.

You also remember what happens to a component of the initial velocity that is perpendicular to the field and the force: nothing! Since there is no component of the acceleration in the direction perpendicular to the field, a velocity component in that direction does not change.

In the gravitational case this is what happens when a ball or other projectile is thrown and we neglect all other forces, such as air resistance. The force and the acceleration are straight down. The horizontal component of the initial velocity does not change. The vertical component changes in accord with the general relation $v = v_0 + at$. This is what we call *projectile motion*. The path of the projectile is a parabola.

The motion of a charge in a uniform electric field is similar. It is in a straight line along the field if there is no initial velocity, or if v_0 is also along the field direction. The path is a parabola if there is a component of the initial velocity that is perpendicular to the field.

If you turn the page through 90°, so that the electric field vector points toward the bottom of the page, the three paths in the figure are just like paths of an object in a gravitational field: the first one for an object thrown straight down, the second one for an object thrown down at an angle, and the third for an object thrown upward at an angle. In each case there is a force and an acceleration in the direction of the field. In the second and third cases there is also a component of the initial velocity (v_{0y} on the figure), which does not change because it is at right angles to the acceleration.



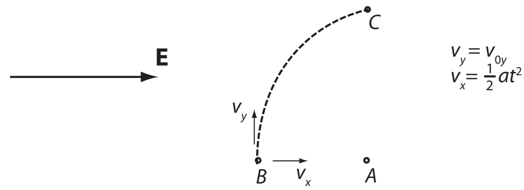
EXAMPLE 5

In the same 5 N/C uniform field as in the previous examples, the $+2 \times 10^{-4}$ C charge is released from rest at B.

(a) Describe its subsequent motion.

(b) What is its kinetic energy when it reaches A?

(c) When the charge is released, as in part (a), it moves toward A. What change would be necessary in the initial motion so that the charge will, instead, move to C? What additional information would be required for a detailed calculation?

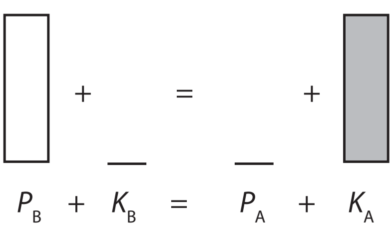


Ans.:

(a) Since the charge is positive, it experiences a force and an acceleration in the direction of the electric field. It starts from rest and moves along the field toward A in accord with the relations that describe the motion of a particle with constant acceleration.

(b) Let the potential and the potential energy of the charge at A be zero. Its potential at B is larger by 15 V. This is the difference in potential energy per unit charge, so that the difference in the potential energy of the charge at the two points is $Q\Delta V = (2 \times 10^{-4}\,\text{C})(15\,J/C) = 3 \times 10^{-3}$ J.

$K_B + P_B = K_A + P_A$, where $K_B = 0$ and $P_A = 0$, so that $P_B = K_A$, i.e., the potential energy at B is transformed to kinetic energy at A, equal to 3 mJ.



(c) If the charge is released from rest in this uniform field, it moves along a field line toward A. To move, instead, to C it has to have an initial velocity component perpendicular to the field. Since there is no field component and no force component in this direction, this velocity component does not change.

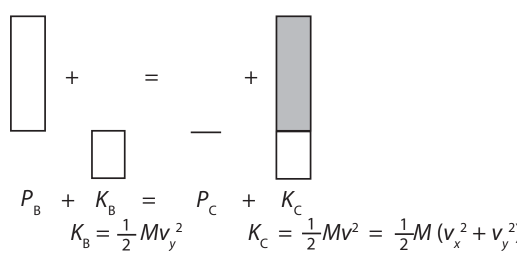
To find this component (v_y) we need to know how long the charge will take to go along the field (in the x direction) from B to A. The presence of a velocity component in the y direction will not change this time. It will take just as long to get to B from C. We can then see how large the y component of the velocity has to be for the charge to go the required distance in the y direction.

To find the time we can use $x = v_{0x}t + \frac{1}{2}at^2$, with $v_{0x} = 0$. Although we know the force, we do not know the acceleration. To go further we need to know the mass of the charged object. Let's say the mass is 5 g, or 5×10^{-3} kg. Then $a = \frac{F}{M} = \frac{QE}{M} = \frac{10^{-3}\,\text{N}}{5\times10^{-3}\,\text{kg}} = 0.2\,\text{m/s}^2$.

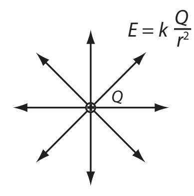
$x = \frac{1}{2}at^2$, so that $t^2 = \frac{2x}{a} = \frac{(2)(3)}{0.2} = 30$, and $t = \sqrt{30} = 5.48$ s.

During that time v_y has to have a magnitude such that the charged object moves the distance from A to C of 4 m. This velocity component is constant (since there is no acceleration in the y direction), so that $y = v_y t$. Hence $v_y = \frac{y}{t} = \frac{4}{5.48} = 0.73$ m/s.

The path from B to C is a parabola. (If you turn the page one-quarter turn clockwise, you see that it looks like the path of a ball moving in a gravitational field.)



Closer to reality: the field of a point charge

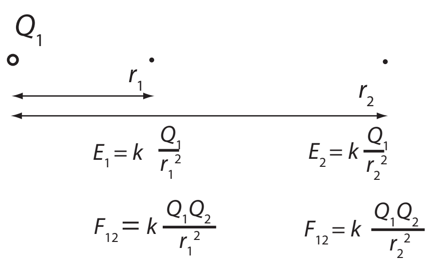


The electric field lines of an individual electron or proton are *radial*. They are straight lines, equally spaced, coming from or going to the charge. This is also true for a uniformly charged sphere, or for any other spherically symmetric charge distribution, as we saw from our discussion of Gauss's law. In the gravitational case we rarely deal with a *point mass*, but stars, planets, and moons are close to being spherically symmetrical.

A radial field is far from uniform. Its magnitude gets smaller as r increases, proportional to $\frac{1}{r^2}$. On the diagram that shows the field lines, the lines are close to each other near Q, and become farther apart as r increases.

We know how the electric potential energy of a system of two positive charges (Q_1 and Q_2) changes when we move Q_2 closer to Q_1: the definition of electric potential energy tells us that the increase in potential energy is equal to the work done *against* the electric force on Q_2. The closer we move Q_2 to Q_1 the more work we have to do against the force of repulsion, and the greater is the resulting potential energy.

When we set out to calculate the work to move Q_2 closer to Q_1 we encounter an obstacle: the field and the force are not constant. We remember that work equals force times displacement, but if the magnitude of the force changes, what are we to do?



The magnitude of the field created by Q_1 is $k\frac{Q_1}{r^2}$, and the magnitude of the force on Q_2 is $k\frac{Q_1Q_2}{r^2}$. We want to calculate the work done as we move the charge Q_2 closer to Q_1, from a distance r_2 to a smaller distance, r_1.

The force changes as r changes and the charge moves through the distance $r_2 - r_1$. We can't multiply the expressions for the force and the distance because we don't know what value to use for F.

This is the kind of problem for which the calculus was invented. It allows us to deal with variables that change, so that we can calculate force times distance even when the force changes.

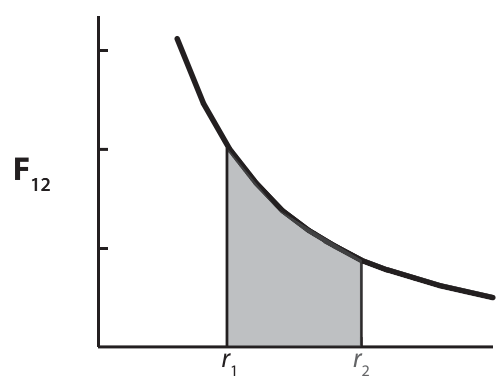
Approximate method, right result

There have been many attempts to calculate the potential energy of two charges with only algebraic methods. We will look at one of the simplest.

Since r varies from r_2 to r_1, r^2 will vary from r_2^2 to r_1^2. The force, F_1, on Q_2 varies from $k\frac{Q_1Q_2}{r_2^2}$ to $k\frac{Q_1Q_2}{r_1^2}$. To get a value between the two we might take one value from each end, using r_1r_2 instead of r^2 in the denominator. This looks like a rough approximation, but it turns out, surprisingly, to give the correct answer. The force is then $k\frac{Q_1Q_2}{r_1r_2}$ and the distance is $r_2 - r_1$. The work done against the repulsion is therefore $(k\frac{Q_1Q_2}{r_1r_2})(r_2 - r_1)$, which is equal to $kQ_1Q_2\frac{r_2-r_1}{r_1r_2}$ or $kQ_1Q_2(\frac{1}{r_1} - \frac{1}{r_2})$.

Let's look more closely at the sign of this result for the work done. If r_1 is less than r_2 then $\frac{1}{r_1}$ is greater than $\frac{1}{r_2}$, and the amount of work is positive. This is as we expect: we have to do (positive) work on the system to move the two positive charges closer to each other.

The point charge, properly



A little calculus, and the use of the expression for the integral of r^n that we cited near the end of Chapter 3, allows us to get away from guesses and approximations. For a constant force the work is equal to the force times the displacement. This is equal to the area under a curve of force against displacement. It is still the area under that curve if the force changes. For a force equal to $k\frac{Q_1Q_2}{r^2}$, and a displacement from r_2 to r_1, the area is equal to the integral $\int_{r_2}^{r_1} k\frac{Q_1Q_2}{r^2}dr$. In the expression for the force, $k\frac{Q_1Q_2}{r^2}$, k and the charges are constant, so that the integral is equal to $kQ_1Q_2\int_{r_2}^{r_1}\frac{1}{r^2}dr$. We only need to evaluate the integral of $\frac{1}{r^2}(=r^{-2})$, i.e., $\int_{r_2}^{r_1} r^{-2}dr$. Since the integral of r^n is $\frac{r^{n+1}}{(n+1)}$, the integral of r^{-2} is $\frac{r^{-1}}{-1}$ or $\frac{-1}{r}$.

If the force moves the charge from r_2 to r_1, we need to evaluate the integral at these two values and calculate the difference, i.e., $\left(\frac{-1}{r_2}-\frac{-1}{r_1}\right)$, or $\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$. We still have to multiply this result by kQ_1Q_2 to get $kQ_1Q_2\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$, the same result as before.

EXAMPLE 6

A fixed charge, Q_1, of 4 μC and a second charge, Q_2, of −3 μC are initially at rest, separated by a distance of 5 cm. The second charge moves toward the first as a result of the Coulomb attraction, until the distance between them is 2 cm. What is then the kinetic energy of the negative charge?

Ans.:
As Q_2 moves toward Q_1, potential energy is lost, and Q_2 gains an equal amount of kinetic energy.

First, calculate the magnitude of the difference in the potential energies. It is $kQ_1Q_2\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$, where $r_1 = 2$ cm and $r_2 = 5$ cm, i.e. $(9\times10^9)(4\times10^{-6})(3\times10^{-6}\left(\frac{1}{2\times10^{-2}}-\frac{1}{5\times10^{-2}}\right) = (0.108)(10^2)$ $(0.5-0.2) = 3.24$ J.

As the charges move toward each other, they lose 3.24 J of electric potential energy and gain the same amount of kinetic energy.

Finding the potential energy and the potential

In the previous section we used the fact that the electric potential energy is equal to the work done against the electric force. Can we also talk about the actual amount of the potential energy, the *absolute* potential energy? Yes, but just as for the gravitational potential energy we need first to choose a reference level where the potential energy is zero.

We know that we can put it anywhere we want, but there is one place that is used most often when we talk about point charges: we let the potential energy, P, be zero when the charges are very far (infinitely far) from each other.

In the calculation of the previous section the two charges are initially separated by the distance r_2. The work done as Q_2 is pushed closer to Q_1, until they are separated by a distance r_1, is $kQ_1Q_2\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$. If we let r_2 become very large, $\frac{1}{r_2}$ becomes very small. When r_2 goes to infinity, $\frac{1}{r_2}$ goes to zero. The work done on Q_2 as it moves from infinitely far away to a distance r_1 from Q_1 becomes $kQ_1Q_2(\frac{1}{r_1})$.

Let's look at the definition again. The increase in potential energy is equal to the work done against the electric force. If we start from the reference level, where $P = 0$, this will also be the potential energy. To bring Q_2 closer to Q_1 means that we have to overcome the electric repulsion between the two positive charges. To bring it from far away to r_1 requires an amount of work equal to $k\frac{Q_1Q_2}{r_1}$, and this is, therefore, their potential energy when they are at the distance r_1 from each other.

We have to be careful with the plus and minus signs. If one charge is positive and the other is negative, for example, the charges attract, and the signs are reversed. There are schemes to keep track of the signs in the formulas, but it is easier first to calculate the magnitude of the work or the change in the potential energy, and to think separately about the signs. Is the work to move a charge against the electrical force positive or negative, i.e., do we have to force them together or hold them apart? Is the value of the potential energy increased or decreased?

Now let's look at the potential. We can again call Q_2 a *test charge*. Take it to be positive. If at a certain point its potential energy is P, the electric potential at that point is $\frac{P}{Q_2}$.

If there are just two charges, Q_1 and Q_2, a distance r_1 apart, then $P = k\frac{Q_1Q_2}{r_1}$, and the potential, a distance r_1 from Q_1, is $k\frac{Q_1}{r_1}$.

We can put our test charge at any point. If its magnitude is Q_2, and the electric force on it is $\mathbf{F}$, then the electric field at that point is $\mathbf{E} = \frac{\mathbf{F}}{Q_2}$. If its electric potential energy at that point is P, then the electric potential at that point is $\frac{P}{Q_2}$.

Starting with the sources

If we know the sources we can start with them. For a point charge, Q, we know that at a distance r from it the electric field is radial, with magnitude $k\frac{Q}{r^2}$, outward if Q is positive and inward if Q is negative. The potential, V, is $k\frac{Q}{r}$, with the sign of Q, positive or negative, depending on which kind of charge Q is. (Here we have taken the reference level $P = 0$ when r is infinitely large and $\frac{1}{r} = 0$.) If Q is not a point charge, but consists of charges distributed with spherical symmetry, these results will still be correct outside the charge distribution. If there are several point charges, we have to take the fields and potentials from each, and add them up. The electric fields are vectors. The potentials are scalars, so that we need only add them up like numbers (which can be positive or negative).

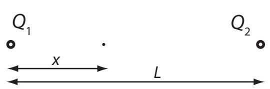
EXAMPLE 7

A charge of $Q_1 = 4\,\mu\text{C}$ is at a distance of $L = 50\,\text{cm}$ from a charge of $Q_2 = -3\,\mu\text{C}$.

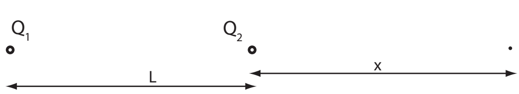
(a) Is there a point on the line between the two charges where the electric potential is equal to zero, and, if so, how far is it from Q_1?

(b) Are there any other points along the line on which the two charges are located where the potential is zero? (To the left of Q_1 or to the right of Q_2.)

Ans.:



Between the two charges, at a distance x from Q_1, the potential is $k\frac{Q_1}{x} + k\frac{Q_2}{L-x}$. It is equal to zero when $\frac{Q_1}{x} = \frac{-Q_2}{L-x}$ or $Q_1(L-x) = -Q_2 x$, i.e., when $(4 \times 10^{-6})(0.5 - x) = (3 \times 10^{-6})(x)$ or $4(0.5 - x) = 3x$. $2 - 4x = 3x$, $7x = 2$, and $x = 0.286$.



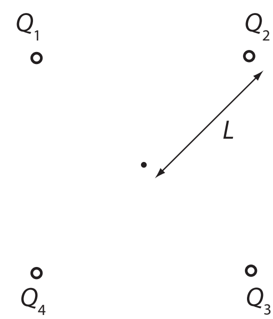
There is a second point where $P = 0$: outside both charges, and nearer to the smaller charge, at a point a distance x from Q_2, where $\frac{Q_1}{L+x} + \frac{Q_2}{x} = 0$. Here $\frac{4}{0.5+x} - \frac{3}{x} = 0$, $4x - 3(0.5 + x) = 0$, or $4x - 1.5 - 3x = 0$, so that $x = 1.5\,\text{m}$.

(Note that there is no corresponding point on the other side of the two charges.)

EXAMPLE 8

Four charges are at the corners of a square. What has to be true for them so that the electric potential at the center of the square is zero?

Ans.:



Let the distance from each of the charges to the center be L, and the charges Q_1, Q_2, Q_3, and Q_4. The potential in the center is $k\frac{Q_1}{L} + k\frac{Q_2}{L} + k\frac{Q_3}{L} + k\frac{Q_4}{L}$ or $\frac{k}{L}(Q_1 + Q_2 + Q_3 + Q_4)$. If this is to be zero, $(Q_1 + Q_2 + Q_3 + Q_4)$ must be zero, i.e., there must be just as much positive as negative charge at the corners, but otherwise it doesn't matter what the charges are. (Of course the electric field will be different for each charge configuration. Under what conditions will it be zero at the center?)

Moving charges

We know the advantages of using energy concepts over those of force whenever we can. Let's see how we can use energy concepts when we talk about charges moving in an electric field. Look at a charge, Q, at a point where the potential is V_1, so that its electrical potential energy is $P_1 = QV_1$. It moves to a point where the potential is V_2 and its potential energy is QV_2. Its kinetic energy at the first point is K_1 and at the second point it is K_2. If there are no other energies that we need to consider, then

$$P_1 + K_1 = P_2 + K_2, \text{or}$$

$$QV_1 + \frac{1}{2}mv_1^2 = QV_2 + \frac{1}{2}mv_2^2$$

To use these relations we don't need to know anything about where there are other charges or what the electric fields look like!

EXAMPLE 9

Bombardment with alpha particles (discovery of the nucleus)

The bombardment of a gold foil with alpha particles from a radioactive source was a groundbreaking experiment by Ernest Rutherford in 1911. The result that the alpha particles rebounded showed that the positive charge of the gold atom was concentrated in a very small region. In fact, the experiment could not distinguish it from a point charge. This was the first demonstration of the existence of the atomic nucleus.

An alpha particle consists of two protons and two neutrons. It has a charge of $+2e$ or 3.2×10^{-19} C and a mass of 6.65×10^{-27} kg.

An alpha particle has an initial kinetic energy of 4 MeV. (1 MeV $= 1.6 \times 10^{-13}$ J.) It is shot toward the center of a gold nucleus ($Z = 79$, i.e., a charge of $79e$).

(a) What is the subsequent motion of the alpha particle?

(b) Draw a graph of the potential energy as a function of the distance between the alpha particle and the gold nucleus. On the same graph draw a line that represents the total energy of the alpha particle. Show the distance of closest approach, D, on the graph.

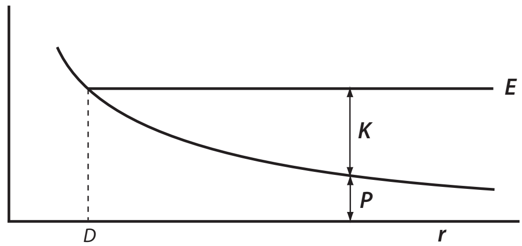
(c) Calculate the distance of closest approach.

Ans.:

(a) Let the system of the alpha particle and the nucleus have zero potential energy when they are very far from each other and the kinetic energy is 4 MeV. The total energy is then 4 MeV.

As the alpha particle moves toward the gold nucleus, it loses kinetic energy and gains potential energy because of the repulsion between the two positively charged objects. It continues to gain potential energy until all of its kinetic energy has been transformed into potential energy. It is then momentarily at rest at the distance of closest approach (D) before rebounding and moving away from the gold nucleus. (Note that when the alpha particle gets close to the nucleus the electrons of the gold atom are far away, and we are ignoring them.)

(b)



(c) If we call the initial point (far away) A and the point of closest approach B, $P_A = 0$, $K_B = 0$, and $K_A = 4$ MeV. Since $P_A + K_A = P_B + K_B$, the potential energy at the point of closest approach is $P_B = K_A = 4$ MeV. We know that $P_B = \frac{kQ_1Q_2}{D}$, and we can now find $D = \frac{kQ_1Q_2}{P_B} = \frac{(9\times10^9)(79\times1.6\times10^{-19})(2\times1.6\times10^{-19})}{4\times1.6\times10^{-19}}$, which is equal to 5.69×10^{-8} m or 56.9 nm.

EXAMPLE 10

Here is a similar example, but in a gravitational field.

A rocket is launched, straight up, from the surface of the earth, with a velocity v_e. Assume that it has this velocity right after launch, and that the engines then shut off so that there is no further energy input. It moves away from the earth and eventually is so far from the earth that the gravitational potential energy of the earth-rocket system is zero. (Using the usual reference level $P = 0$ when $\frac{1}{r} = 0$.) Neglect any forces other than the gravitational force of the earth.

(a) The minimum velocity that the rocket must have at the beginning to get that far is called the escape velocity. How large is that velocity, v_e?

(b) Draw a graph of the gravitational potential energy P as a function of the distance r. Mark on it the total energy, E, the initial kinetic energy, $\frac{1}{2}mv_e^2$, and the radius of the earth, R_e, and on it show the kinetic energy K of the rocket.

Ans.:

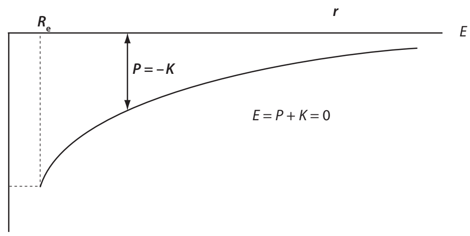
(a) The rocket starts out with a potential energy $P = -G\frac{mM_e}{R_e}$, where m is the mass of the rocket and M_e and R_e are the mass and the radius of the earth. Its initial kinetic energy is $\frac{1}{2}mv_e^2$.

At its destination $P_f = 0$. If it just gets there, without energy to spare, its kinetic energy, K_f, is also zero. Since both P_f and K_f are zero, so is the total energy.

If there is no energy input, this is also the energy at the start, i.e., $-G\frac{mM_e}{R_e} + \frac{1}{2}mv_e^2 = 0$ or

$\frac{1}{2}mv_e^2 = G\frac{mM_e}{R_e}$, so that $v_e = \sqrt{\frac{2GM_e}{R_e}}$, which we see to be independent of the mass of the rocket and equal to $\sqrt{\frac{(2)(6.67\times10^{-11})(5.97\times10^{24})}{6.38\times10^6}}$, which is equal to 1.12×10^4 m/s, or close to 7 miles per second.

(b)



High-speed electrons are used in many devices. In an *X–ray tube*, for instance, they hit a "target" from which the X–rays are emitted.

Another application is the vacuum tube rectifier or *diode*. It contains two metal *electrodes*. One of them (the *cathode*) is heated, so that some of its electrons are given enough energy to leave it. The other one (the *anode*) is at a higher potential, so that the electrons liberated at the cathode are accelerated toward it. Since the anode is not heated, electrons cannot leave it, and there is current in only one direction. If the electrons start with negligible kinetic energy at the cathode, they will arrive at the anode with a kinetic energy equal to $Q\Delta V$ joules, where ΔV is the potential difference between the anode and the cathode.

If the anode has a hole in it, or is in the shape of a ring, some of the electrons, instead of slamming into it, will continue to the region on the other side of the anode where the electric field is zero, or at least quite small. Such an arrangement is called an *electron gun*.

These applications are so important that they led to the introduction of the electron volt as a unit of energy. We have used it before, and it is used universally in the discussion of atomic, molecular, and nuclear energies. One electron volt is equal to 1.6×10^{-19} J. Hence electrons accelerated by a potential difference of ΔV volts will gain a kinetic energy of ΔV electron volts.

EXAMPLE 11

In a vacuum tube diode the anode voltage is 60 V. The cathode voltage is zero, and the electrons leave the cathode with negligible energy.

(a) What is the kinetic energy of the electrons when they arrive at the anode? Give the answer in joules and in electron volts.

(b) What is the speed of the electrons at the anode?

Ans.:

(a) At the cathode the potential energy of the electrons, P_C, is zero, and so is their kinetic energy, K_C. At the anode the potential energy is $P_A = QV = (-1.6\times10^{-19})(60) = -9.6\times10^{-18}$ J. Since $P_C + K_C = P_A + K_A$, $0 = K_A + P_A$, and $K_A = -P_A = 9.6 \times 10^{-18}$ J or 60 eV.

(b) $K = \frac{1}{2}mv^2$ so that $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(9.6\times10^{-18})}{0.91\times10^{-30}}} = 4.6 \times 10^6$ m/s.

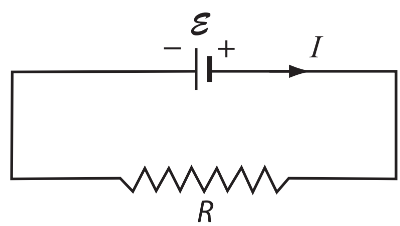
9.2 Energy transformations and electric circuits

Battery and resistor

An electric battery is a device that transforms stored internal ("chemical") energy to electric potential energy. The chemical reactions inside it produce a difference of electric potential between its two terminals. We can use this energy by changing it into other kinds of energy when we make the battery part of an electric circuit.

Here is the simplest circuit: we attach a wire to the battery between its positive and negative terminals. As a result of the potential difference electrons move in the wire. There is now a *current* in the wire.

Here is a "schematic" diagram to represent this circuit.



The symbol in the top part with the + and − signs represents the battery. The symbol with the lines going back and forth in the bottom part represents the wire. Its shape is meant to indicate that there is quite a bit of it, as in a coil of wire. It is called a *resistor* and is marked R. The straight lines between the symbols for the battery and the coil of wire are there only to connect them.

Since the electrons are negatively charged, they move through the wire from the battery's negative terminal toward the positive one. However, as an accident of history, the electric *current* is defined as the amount of positive charge passing a place in the wire in a second. If the charges that move are actually negative (as is most often the case, since it is the electrons that move in a wire), the direction of what we call the current is opposite to the direction of motion of the charges. In other words, a current to the right can consist of positive charges going to the right or of negative charges going to the left. The symbol I is used for the current, and its SI unit is the coulomb/second, which is also called the *ampere*.

What happens in the wire?

You might be surprised that the current is the same in every part of the circuit shown in the figure. In fact, you should be. There is an electric field in the metallic wire, from the positive terminal toward the negative terminal of the battery, just as there is from any positive charge toward a negative charge. Aren't the electrons accelerated? Wouldn't that make the current larger and larger along the direction of motion?

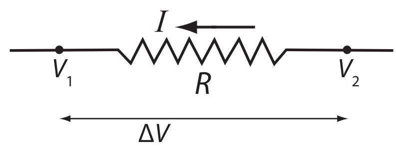
That's what would happen if the wire were an empty tube. But the wire is made of ions (the parts of the atoms that are left behind when the moving "free" electrons are detached). The electrons bounce along among them, making collisions along the way that slow them down.

Let's see what happens to the electrons. After each collision an electron is scattered and goes in some new direction. This new direction can be at any angle, and all directions are equally probable. For all of the scattered electrons, going in all of the equally probable directions, the average velocity is zero. We can look at a single electron that represents the average behavior of all of them. It is as if this electron were stopped by the collision. It is again accelerated, but has to start over after each collision. The result is that the average velocity of the electrons is constant, and the current is the same in each part of the wire.

The situation is somewhat analogous to that of falling raindrops. If they fell through empty space they would be accelerated by the gravitational force. (It would be very dangerous to be outside and to be bombarded by them!) Instead they collide with air molecules and move much more slowly than they would in empty space and with constant velocity.

When the electrons collide with the ions they slow down and give up some of their kinetic energy. The energy is shared by the ions and is now internal energy of the wire. This increased internal energy is perceived by us as a higher temperature.

Look at a coil of wire. There is a difference between the values of the electric potential at the two ends of the coil. This potential difference across the coil, $\Delta V = V_2 - V_1$, divided by the current, I, through the coil, is called the coil's *resistance*, R, equal to $\frac{\Delta V}{I}$. The SI unit for the resistance, equal to the volt/ampere, is called the ohm, with the symbol Ω (Greek capital *omega*).

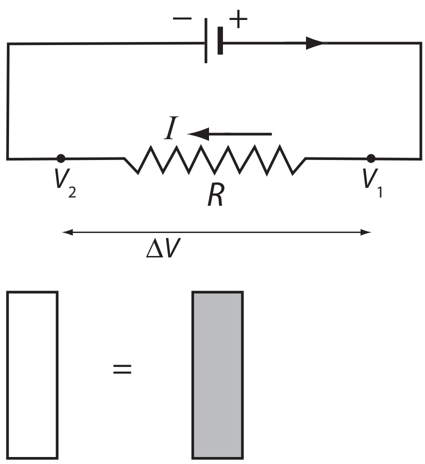


The electrons carry electric potential energy from one part of the circuit to another. In the battery they get energy from the internal energy of the battery materials. In the resistor they give up energy to the material of the wire through collisions with the metal ions. This energy becomes internal energy of the wire.

The potential difference, or *voltage difference*, ΔV, represents a difference in potential energy per coulomb. A volt is a joule/coulomb. The current is I amperes, or I coulombs/second. $I\Delta V$ is therefore the number of joules per second, or the power in watts, transformed in the wire from electric potential energy to internal energy of the wire.

The resistance of a wire varies with temperature, but is otherwise constant. It is a property of the wire. This is so for metals, but not for all other materials. When the resistance is constant (at a given temperature), so that it does not depend on the current, the material is said to follow *Ohm's law*.

The resistance of metal wires used for connections in circuits is usually so small that it can be neglected. On the other hand, the energy



$$Q\mathcal{E} = Q\Delta V$$

$$\mathcal{E} = IR$$

dissipated in the wire ("filament") of a light bulb is so large that the wire gets white hot, and part of the transformed energy is radiated away as visible light.

One more new term: the amount of energy that is transformed in the battery (or other device) from some other kind of energy to electric potential energy divided by the charge that passes through it is called its *emf*. A 9 volt battery has an emf of 9 volts. For each coulomb that moves through it, 9 joules of energy are transformed from stored chemical energy to electric potential energy.

"Emf" stands for *electromotive force*, but physicists tend to be reluctant to write this out, because it is not a force at all, but an energy divided by a charge.

EXAMPLE 12

Go to the PhET website (http://phet.colorado.edu) and open the simulation *Ohm's Law*. Set R at 140 Ω by dragging the resistance button up and down.

(a) Change the voltage while R remains constant. How do V and I change with respect to each other?

(b) Change R while V remains constant. How do I and R change with respect to each other?

Set V at 4 V. Set R at 400 and 600 Ω. For each case calculate I and compare the result to the value on the screen.

Go to the PhET website and open the simulation *Battery–resistor Circuit*. Uncheck the three buttons on the right. Set $V = 12.00\text{V}$ and $R = 0.4\,\Omega$.

Calculate the current and compare it to the value on the ammeter.

Click on "show inside battery" to see the electrons being pushed (by little elves?).

Click on "show cores" to see the ions. Reduce the voltage and observe the change in the motion of the ions and in the temperature.

EXAMPLE 13

In a flashlight a 1.5-V battery is connected to a light bulb.

(a) How much energy is given to each electron as it passes through the battery? Where does this energy come from?

(b) What happens to the electric potential energy of the electrons as they pass through (i) the wire and (ii) the bulb?

Ans.:

(a) Each electron gets an amount of energy, $Q\Delta V$, whose magnitude is $(1.6 \times 10^{-19})(1.5) = 2.4 \times 10^{-19}$ J. This energy is transformed from the battery's internal energy to electric potential energy.

(b) (i) If you assume that the wire has no resistance there is no difference of potential across the wire and no energy transformation. (ii) The full potential difference is then across the bulb, and in the bulb some of this electric potential energy is transformed to internal energy (thermal energy) of the filament and some is radiated away as visible light.

EXAMPLE 14

A 25 Ω resistor is connected across the terminals of a battery whose emf is 6 V.

(a) Draw a schematic diagram of the circuit.

(b) What is the potential difference across the resistor?

(c) What is the current through the resistor?

(d) What is the current through the battery?

(e) What is the power generated by the battery?

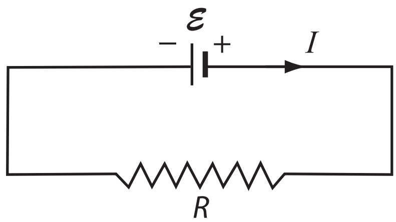
(f) What is the power dissipated in the resistor?

(g) What is the energy (in joules) transformed in the resistor in one minute?

(h) What is the amount of energy in kilowatt-hours (*kwh*) generated by the battery in one day?

Ans.:

(a)



(b) The potential difference across the resistor has the same magnitude as the potential difference across the battery. The connecting lines in the schematic diagram are assumed to have no resistance, so that there is no potential difference across them. ($\Delta V = IR = 0$ for them.) In an actual circuit the wires have resistance, but it can usually be neglected because it is much less than the resistance of the resistors.

(c) $I = \frac{\Delta V}{R} = \frac{6}{25} = 0.24\,\text{A}$.

(d) The current is the same in every part of this circuit, 0.24 A.

(e) Power = $I\Delta V$. For the battery it is equal to $\mathcal{E}I = (0.24)(6) = 1.44\,\text{W}$.

(f) $I\Delta V = 1.44\,\text{W}$.

(g) The power is the energy transformed per second. In 60 s the energy transformed is therefore $(60\,\text{s})(1.44\,J/s) = 8.64\,\text{J}$.

(h) In one day there are (24)(3600) seconds. The energy generated in a day is therefore $(1.44)(24)(3600)\,\text{J} = 1.24 \times 10^5\,\text{J}$.

A *kilowatt-hour* is the energy transformed in an hour if the rate is 1000 W. It is $(3600)(1000)\,\text{J}$ or $3.6 \times 10^6\,\text{J}$. $1.24 \times 10^5\,\text{J} = \frac{1.24\times10^5}{3.6\times10^6} = 0.035\,\text{kwh}$.

EXAMPLE 15

Two buzzers (which can be treated as resistors) are connected, end to end, to a battery. When resistors are connected so that there is the same current in each, they are said to be connected *in series*. The emf of the battery is 12 volts. The resistors are $R_1 = 6\,\Omega$ and $R_2 = 9\,\Omega$.

(a) Draw a schematic diagram for this circuit.

(b) What is the resistance of a single resistor that could replace the two resistors without changing the current through the battery? This is called the "equivalent resistance," R_{eq}.

(c) What are the potential differences across each of the two resistors in the original circuit?

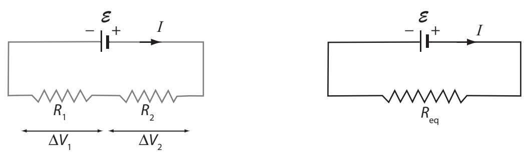
(d) What is the power dissipated in each resistor?

(e) What is the power dissipated in R_{eq}?

(f) What is the power generated in the battery?

Ans.:

(a)



(b) The voltages ΔV_1 and ΔV_2 add up to the voltage ΔV across the equivalent resistance. $\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$. $R_{\text{eq}} = \frac{\Delta V}{I} = R_1 + R_2$.

$R_{\text{eq}} = R_1 + R_2 = 6 + 9 = 15\,\Omega$.

(c) $\Delta V_1 = IR_1$.

$I = \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{12}{15} = 0.8\,\text{A}$.

$\Delta V_1 = (0.8)(6) = 4.8\,\text{V}$.

$\Delta V_2 = (0.8)(9) = 7.2\,\text{V}$.

Note that $\Delta V_1 + \Delta V_2 = 12\,\text{V} = \mathcal{E}$.

(d) $P_1 = I\Delta V = I^2R_1 = (0.8)(4.8) = 3.84\,\text{W}$.

$P_2 = I^2R_2 = (.64)(9) = 5.76\,\text{W}$.

$P_1 + P_2 = 9.60\,\text{W}$.

(e) $P = I^2R_{\text{eq}} = (.64)(15) = 9.60\,\text{W}$.

(f) $P = \mathcal{E}I = (12)(.8) = 9.60\,\text{W}$.

EXAMPLE 16

Two resistors are connected separately across the battery. When resistors are connected so that the potential difference is the same through each, they are said to be connected *in parallel*. The emf of the battery is 12 volts. The resistances of the two resistors are $R_1 = 6\,\Omega$ and $R_2 = 9\,\Omega$.

(a) Draw a schematic diagram for this circuit.

(b) What is the current through each resistor?

(c) What is the current through the battery?

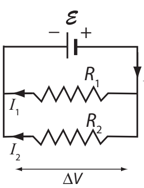
(d) What is the resistance of a single resistor that could replace the two resistors without changing the current through the battery? This is the equivalent resistance for the combination.

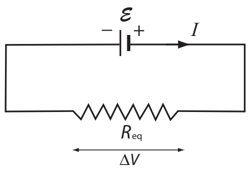
(e) What is the power dissipated in each resistor?

(f) What is the power dissipated in the equivalent resistance?

(g) What is the power generated by the battery?

Ans.:
(a)





(b) $I_1 = \frac{\Delta V}{R_1} = \frac{12}{6} = 2\,\text{A}$.
$I_2 = \frac{\Delta V}{R_2} = \frac{12}{9} = 1.33\,\text{A}$.

(c) The currents I_1 and I_2 come together to form the current I through the battery: $I = I_1 + I_2 = 2 + 1.33 = 3.33\,\text{A}$.

(d) If there were just a single resistor, R_{eq}, across the battery, its resistance would be $\frac{\mathcal{E}}{I}$ or $\frac{\Delta V}{I}$. Here the potential difference across each resistor is the same, but the current through each is different. (For the series connection of the previous example the current through each resistor is the same, but the two voltages are different.)

$$R_{eq} = \frac{\mathcal{E}}{I} = \frac{12}{3.33} = 3.60\,\Omega.$$

We could also do the calculation more generally: $I = \frac{\Delta V}{R_{eq}} = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$ or $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$.

(e) $I_1 \Delta V = (2)(12) = 24\,\text{W}, I_2 \Delta V = (1.33)(12) = 16\,\text{W}$.

(f) $I^2 R_{eq} = (3.33)^2(3.60) = 40\,\text{W}$. [This is the sum of the two values calculated in part (e).]

(g) $\mathcal{E} I = (12)(3.33) = 40\,\text{W}$.

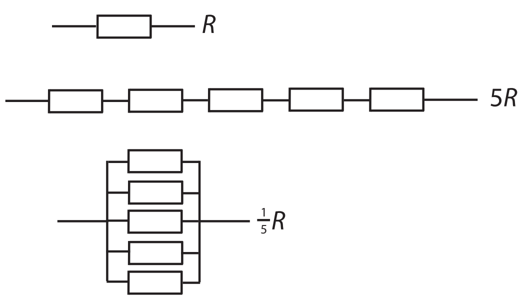
When lights, toasters, vacuum cleaners, and other appliances are "plugged in" at home, they are connected in parallel. That allows each of them to be connected independently. The "electric meter" measures the amount of energy that is used, in kilowatt–hours, and this is what shows up in the monthly bill.

Resistivity: separating out the property of the material

The resistance of a wire depends on its shape, i.e., its length and cross-sectional area, and on the material of which it is composed. Can we separate the effect of the material's properties from those of the wire's size?

Just think of the wire as a number of pieces in series. Suppose there are five pieces, each with a length L and a resistance R. The resistance of the whole wire (its equivalent resistance) is $5R$, and the length is $5L$. We see that the resistance is proportional to the length.

Similarly we can think of the wire as made up of five pieces in parallel, each with resistance R and cross-sectional area A. The resistance of the whole wire (its equivalent resistance) is then $\frac{R}{5}$. The total area is $5A$. We see that the resistance is inversely proportional to the area.



In general then we can write $R = \rho \frac{L}{A}$, showing the length and area dependences. The remaining factor, ρ (Greek *rho*), depends only on the material of the wire and its temperature. It is called the *resistivity*.

EXAMPLE 17

Go to the PhET website and open the simulation "Resistance in wire." Vary ρ, L, A in turn to see the effect on R. Choose values of ρ, L, and A and compare them to the values on the screen.

EXAMPLE 18

What is the diameter of a 2-cm-long tungsten filament in a "60 W" incandescent light bulb? (At the operating temperature the resistivity is about $10^{-6}\,\Omega\,\text{cm}$.)

Ans.:
Since the power is $P = IV$, or $\frac{V^2}{R}$, we can write $R = \frac{V^2}{P} = \frac{110^2}{60} = 201.7\,\Omega$. Since $R = \rho\frac{L}{A}$, we can write $A = \frac{\rho L}{R} = \frac{(10^{-6})(0.02)}{201.7} = 9.9 \times 10^{-11}\,\text{m}^2$. $A = \frac{\pi}{4}D^2$, so that $D^2 = 1.26 \times 10^{-10}$ and $D = 11 \times 10^{-6}\,\text{m} = 11\,\mu\text{m}$.

9.3 Summary

Like the gravitational potential energy the electric potential energy has two great advantages. One is that it is a scalar quantity. That makes it much simpler to use than the gravitational and electric forces, which are vector quantities. The other is that when a particle moves from one point to another, the difference in potential energy depends only on the potential energies at the starting point and at the end point, and not on the particular path between the two.

When a charge moves in an electric field the increase in electric potential energy is equal to the work done against the electric field.

The change in the potential energy of a charge, divided by the magnitude of the charge, is called the change in the electric potential.

No work is done when a charge moves along an equipotential.

In an electric field the acceleration of a charge is along the electric field line. In a uniform field, if the initial velocity of the charge is zero or along the field, the charge moves along the field line. If the charge has an initial velocity that is not along the field it moves in a parabola. The velocity component parallel to the field changes, but the component perpendicular to the field remains constant.

The electric potential energy of a system of two point charges is $k\frac{Q_1Q_2}{r}$, where the reference level is taken to be at $r = \infty$ ($\frac{1}{r} = 0$). With this reference level the potential at a distance r from a point charge is $k\frac{Q}{r}$.

In our homes and in our other surroundings, electric circuits are everywhere. They can have many purposes, but their basic property is that wires allow the transport of charges (*currents*), and with them the transport of energy. The simplest circuits consist of batteries and resistors.

The current in a wire is the charge passing a cross section per second.

The resistance of a wire is the potential difference across the wire divided by the current through the wire. $R = \frac{\Delta V}{I}$.

If R is constant when ΔV and I vary, the wire is said to follow Ohm's law. (R can and usually does vary with temperature.)

When a current passes through a resistor some electric potential energy is changed to internal energy. The amount of power (= energy per second) that is transformed is $I\Delta V\left[= I^2R = \frac{(\Delta V)^2}{R}\right]$ watts.

When two resistors are connected so that the same current passes through both, they are said to be connected in series. The equivalent resistance of a combination of resistors in series is their sum. $R_{\text{eq}} = R_1 + R_2 + R_3 + \cdots$.

When two resistors are connected so that the potential difference across each is the same, they are said to be connected in parallel. The reciprocal of the equivalent resistance of a combination of resistors in parallel is the sum of their reciprocals. $\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$.

The resistivity, ρ, is a property of the material of which a wire is made. A wire of length L and cross-sectional area A has a resistance of $\rho\frac{L}{A}$.

9.4 Review activities and problems

Guided review

1. The electric field in a region is uniform, in the y direction, with magnitude 12 N/C. Use the point A (0,0) as the reference point where $V = 0$. For each of the points $B(3,0)$, $C(0,3)$, and $D(3,3)$ (where all distances are in meters):

(a) What is the force on a $+3 \times 10^{-5}$ C charge at B, C, and D?

(b) How much work needs to be done to move this charge from the origin to B? to C? to D?

(c) What is the electric potential energy of the system when the charge is at B? at C? at D?

(d) What is the electric potential at B? at C? at D?

(e) How do the answers to the previous parts change if the charge is negative instead of positive?

2. Go to the PhET website and open the simulation *Charges and Fields*. Select "grid." Select "show numbers" to see the length scale. Put four positive charges at the corners of a square with sides 1 m. Deselect "show numbers."

(a) With the blue voltage sensor explore the equipotentials (i) close to the charges and (ii) outside the square and in the intermediate regions.

What is the shape of the equipotentials as you come close to one of the charges? Why?

(b) Use an "E-field sensor" to check what you know to be the magnitude of the field in the center. Explore the various regions with this sensor.

(c) Plot the equipotentials outside the square at regular intervals from one of the charges. What happens to the shape of the equipotentials as you move away from the square? Why?

(d) "Clear All" and replace the two charges on the right with negative charges. Plot equipotentials near to and far from the charges. What is the direction of the field at the center this time? What is it along the horizontal and vertical lines through the center?

3. A positive charge of $3\,\mu$C is at the origin. A negative charge with the same magnitude is 8 m away along the x direction.

(a) What are the magnitude and direction of the electric field at the points between the two charges, 2, 4, and 6 m from the positive charge?

(b) What is the electric potential at each of these three points? (The reference point, where $V = 0$, is infinitely far away.)

4. In the field of problem 1, how much work is necessary to move an electron from C to D, from C to A, and from C to B?

5. (a) In the field of problem 1, what is the path of an electron released from rest at point C?

(b) How long will it take the electron to reach point A?

(c) The electron has just the right initial velocity at C so that its path will take it to B. How long will it take for the electron to reach B?

6. A proton is fixed at a point. A second proton is released from rest at a point 2 nm from the first proton. What is its kinetic energy when it is 4 nm from the first proton?

7. Four charges, $+5\,\mu$C each, are at the corners of a square whose sides are 2 m. What is the potential at the center of the square?

8. Five identical charges are at the points of a symmetrical five-pointed star, so that the distances from each charge to its neighbor are the same and the distances from each to the center are the same. Is it possible for the electric potential at the center to be zero? If yes, what must be true? If not, why not?

9. A proton with initial kinetic energy 2 MeV is shot straight at an alpha particle.

(a) Describe the motion of the proton.

(b) Draw a graph of the electric potential energy of the proton as a function of its distance from the α particle. Show the distance of closest approach on the graph.

(c) Calculate the distance of closest approach.

10. A rocket is launched straight up at twice the escape velocity. How fast is it moving when the gravitational potential energy is zero?

11. An electron, a proton, and an alpha particle are accelerated through the same potential difference of 50 V. What are the resulting kinetic energy and speed of each?

12. A battery with an emf of 20 V is connected to a lightbulb whose filament has a resistance of 400 Ω.

(a) What is the current in the circuit?

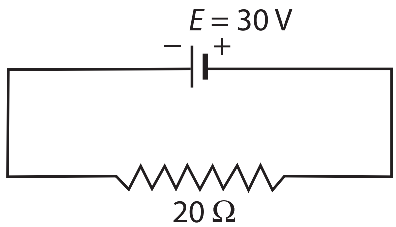
(b) The resistance is doubled by adding a second, equal, bulb to the circuit (in series). What is the current now?

(c) What assumptions did you have to make to get the results for parts (a) and (b)?

(d) An ammeter is added to the circuit to measure the current. It reads 0.020 A. This is not the current that you calculated in part (b). What can you conclude from this reading about the ammeter? What should you look for in a different ammeter to get a result closer to that of your calculation in part (b)?

13. A 9-V battery supplies a lamp with a current of 5 mA. How much energy does the lamp use in 10 s?

14.



(a) What is the voltage across the resistor?
(b) What is the current through the resistor?
(c) In what direction does the current flow (clockwise or counterclockwise)?
(d) In what direction do the electrons move (clockwise or counterclockwise)?
(e) How much energy is supplied by the battery in each second?

15. Three resistors, 2 Ω, 4 Ω, and 6 Ω, and a battery whose emf is 24 V are connected in series.
(a) What is the potential difference across each of these four elements?
(b) What fraction of the total power is dissipated in each of the resistors?
(c) What energy transformation takes place in each of the four circuit elements?

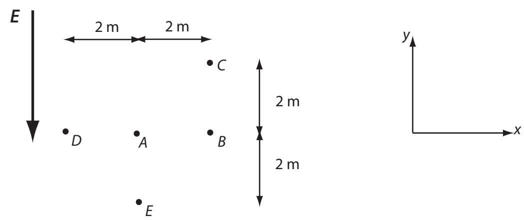
16. Two electronic devices are separately plugged in at home. Each has a resistance of 60 kΩ. (Do the problem as if the source of emf was DC.)
(a) Are they connected in series or in parallel?
(b) What is the current in each?
(c) How much energy is used by each in one second?
(d) Find the equivalent resistance of the two devices.
(e) What is the power supplied by the source of the emf?
(f) How long would both have to be connected to use 1 kw-h of energy?

17. A wire has a resistance of R Ω. A second wire (of the same material) has half the length of the first and twice its diameter. What (in terms of R) is the resistance of the second wire?

18. How long must a copper wire ($\rho = 1.7 \times 10^{-8}\ \Omega$ cm) of diameter 0.2 mm be to have a resistance of 1 Ω?

Problems and reasoning skill building

1.



There is a uniform electric field of 3 N/C in the (−y) direction, as in the figure. Take the point A as a reference, with $V = 0$. What is the potential at points B, C, and E?

2. An electron is released from rest at point A in the field of the previous question.
(a) What is its path?
(b) What is its kinetic energy (in J and in eV) after it has traveled 4 m?

3. Can you put charges at the corner of a square so that both the electric field and the electric potential at the center (with the usual reference at infinity) are equal to zero?

4. An electron passes a point at which the potential is 5 V, with a kinetic energy of 7 eV. Some time later it passes a point where the potential is 8 V. What is its kinetic energy there, in eV and in J?

5. A radio draws a current of 150 mA when plugged into a wall socket with a voltage of 120 V. How much power does the radio use?

6. (a) A circuit consists of two similar lamps in series across a 110 V source of emf. The current through one lamp is 1 A. What is the current through the other lamp?
(b) A circuit consists of two similar lamps in parallel across a 110 V source of emf. The current through one lamp is 1 A. What is the current through the other lamp?
(c) What is different about the lamps in parts (a) and in (b)? (Give a quantitative answer.)

7. The resistance between two points in a cell is $1.6 \times 10^{10}\ \Omega$.

(a) There is a potential difference of 60 mV between the points. What is the current?

(b) The current consists of singly charged Na^+ ions ($Q = +e$). How many ions flow past each point in 0.3 s?

8. A toaster has a resistance of $22\ \Omega$. It takes 2 min for one slice of toast. What is the energy that is used by the toaster in that time?

9. You have a battery and some bulbs, and you make three circuits. Rank the circuits in order of the largest current to the smallest, and in order of the brightest to the dimmest.

(a) one bulb only;
(b) two bulbs in series;
(c) two bulbs in parallel.

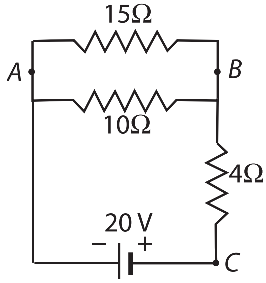
10. A proton is shot directly at a nucleus of sulfur ($Z = 16$). If it gets to within about 7×10^{-15} m of this nucleus it will be so close as to be within the range of the nuclear force, and can initiate a nuclear reaction. What initial kinetic energy (in MeV) must the proton have to achieve this?

11. In a uniform electric field of 5 N/C in the y direction, an electron is given an initial kinetic energy of 12 eV in the x direction.

(a) Sketch the path of the electron on a diagram, on which you also show E and the initial velocity, v_0.

(b) Later the electron passes a point where the value of y has changed by 5 cm from where it started. What is the potential at that point? What is the electron's kinetic energy there? Take the potential to be zero at the electron's initial position.

12.



(a) What is the equivalent resistance of the $15\ \Omega$ and the $10\ \Omega$ resistors between points A and B?

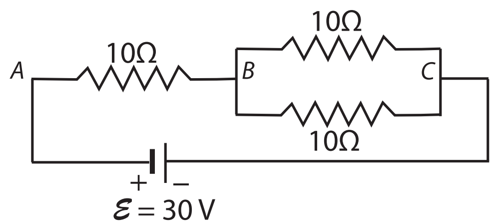
(b) What is the equivalent resistance of the three resistors between points A and C?

(c) What is the current through the battery?

(d) A wire (with zero resistance) is connected between points A and B. What is the current through the battery now?

(e) Why would the analysis that you used for part (d) fail if the question is instead to find the current when points A and C are connected with a wire? (In other words, which of your assumptions would no longer be tenable?)

13.



(a) What is the equivalent resistance between points B and C? Between A and C?

(b) What is the current in each part of the circuit?

(c) What is the potential difference between points A and B? Between points B and C? A and C?

(d) What is the power dissipated in each resistor? What is the power generated in the battery?

14. A coil whose resistance is $36\ \Omega$ is connected to a source of 120 V and immersed in $\frac{1}{2}\ell$ ($=500\ \text{cm}^3$) of water at 20°. Assume that all of the energy produced in the coil is used to heat the water. How long does it take to bring the water to the boiling point?

15. A 1 cm^3 copper block is used to make a wire whose resistance is $10\ \Omega$. What are the length and diameter of the wire? ($\rho = 1.7 \times 10^{-8}\ \Omega\,\text{m}$.)

Multiple choice questions

1. Two equal light bulbs (assume that they have the same constant resistance) are connected across a constant 110-V source of emf, first in parallel and then in series. The ratio of the power developed in the parallel connection to that in the series connection is

(a) 4
(b) 2

(c) 1
(d) $\frac{1}{2}$
(e) $\frac{1}{4}$

2. The resistance of a light bulb with a metal filament increases as the temperature of its filament changes. If we repeat the previous question, the answer is now
(a) larger
(b) smaller
(c) the same
(d) unknown

3. Adding impurities to a metal increases its resistivity. A certain alloy of copper has twice the resistivity of copper. A length of wire made of this alloy has a resistance R_a and a diameter d_a. A copper wire with the same length and resistance has a diameter d_{Cu}. The ratio $\frac{d_{Cu}}{d_a}$ is
(a) 4
(b) 2
(c) 1
(d) $\frac{1}{2}$
(e) $\frac{1}{4}$

4. A resistor is connected across a battery with constant emf. A second resistor whose resistance is half as large is now connected across the battery in parallel with the first. That changes the total power dissipated by a factor of
(a) $\frac{1}{3}$
(b) $\frac{1}{2}$
(c) 3
(d) 2
(e) 1

5. The resistivity of a metal increases when the temperature increases because of which of the following:
(a) The electrons move faster.
(b) The metal ions vibrate with greater energy.
(c) The electrons collide with each other more often.
(d) More of the electrons become free to move.
(e) More of the metal atoms become ionized.

6. How much current flows in a 6 W clock radio plugged into a household receptacle?
(a) 50 A
(b) 660 A
(c) 50 mA
(d) 18 A
(e) 180 mA

7. A computer is connected across a 120 V source for 10 hours. It requires an average of 0.20 A. The energy that is used is
(a) 240 J
(b) 860 J
(c) 860 kJ
(d) 14.4 kJ
(e) 24 J

Synthesis problems and projects

1. Go to the PhET website and open the simulation *Charges and Fields*. Select "grid" and "show E-field."

(a) Make a horizontal line of eight negative charges, spaced five divisions apart, near the middle of the screen. What happens to the field direction below the line, near its middle, as you add more and more charges?

(b) Make a line of the same number of equally spaced positive charges, parallel to the first, and 10 divisions away. What happens to the field between the lines as you add more and more charges?

(c) Plot a series of roughly equally spaced equipotentials, about half a division apart along a vertical line near the middle of your capacitor. What do they tell you about what the potential and the field are in the capacitor?

Check "clear" on the blue sensor. Check "show numbers" and use the equipotential plotter to measure the potential difference between the two lines of charge. (Stay along a line between the charges.) Calculate the electric field in the middle of the capacitor from the potential difference. Now use an E-field sensor to see what the field is. Are the two at least approximately the same? This is, of course, not a perfect capacitor. What changes would be needed to make it closer to perfect?

(d) Describe the variation of the electric potential and the electric field as you move in directions perpendicular and parallel to the lines of charge between the charged lines.

(e) Compare your observations to your expectations based on the last example in Chapter 8 and Section 9.1.

2. Go to the PhET website and open the simulation *Circuit Construction Kit* (DC only). The

components can be moved with the mouse and connected by placing the red circles on top of each other. The length of the connecting wires can be changed and they can be turned by "pulling" on them. A component can be removed by selecting it and pressing the backspace key on your keyboard. A connection can be broken by clicking on it (a yellow ring appears) and pressing the backspace key.

Construct a circuit consisting of a battery and a light bulb. You can tell whether you have made the connections by seeing whether the electrons move.

Look at "lifelike" and "schematic." You can also hide the electrons (select "Advanced") and make the schematic diagram look more like the usual schematics. Select "show values."

(a) Select "voltmeter." It and its probes can be moved with the mouse. Measure the voltage across the battery, the bulb, and a wire.

(b) Select "ammeter." It can then be moved with the mouse and connected. Connect it in the circuit so that the current goes through it (it replaces a wire). How does the current differ in different parts of this series circuit?

(c) Initially the connecting wires do not have resistance. Select "advanced" to change their resistivity. Set the resistivity slider near the middle and measure the voltages across the battery, the light bulb, and the wires. What is the resistance of a wire that is about two inches long?

3. Go to the PhET website and open the simulation *Circuit Construction Kit*. (See the more detailed instructions in the previous question.)

(a) Construct a circuit with two light bulbs in series with a battery and an ammeter. Measure the voltage across each bulb and across the battery. How are they related? What is the resistance of each bulb?

(b) Construct a circuit with two light bulbs in parallel, connected to a battery. How many ammeters do you need to measure all of the currents at the same time? Measure the voltage across each bulb and across the battery and all of the currents. How are the currents related?

(c) Calculate the equivalent resistance of each circuit. Which circuit has the greatest equivalent resistance?

(d) Calculate the power dissipated in each circuit. Which circuit uses the largest power?

(e) What happens in each of the circuits when you unscrew a bulb? You can incorporate a switch next to each bulb to disconnect it from the battery.

(f) Are the lights in a house connected in series or in parallel? Explain.